Limits

- calculus is the mathematics of change
  - as the amount of change gets smaller and smaller, we use limits to determine values at given points

EX → Distance travelled by an object at time t is given by $f(t)$ (t in sec, $f(t)$ in ft).

To determine average speed of object, we would use $\frac{d}{t}$, or $\frac{\Delta f(t)}{\Delta t}$.

Let $f(t) = 4t^2$. If we wanted to find the average speed from $t=0$ to $t=3$,

$$\frac{\Delta f(t)}{\Delta t} = \frac{f(3)-f(0)}{3-0} = \frac{4(3)^2-4(0)^2}{3} = \frac{36-0}{3} = 12 \text{ ft/s}$$

What about instantaneous speed at $t=3$? Let's say that h is an infinitesimal amount of time after $t=3$. To find the instantaneous rate,

$$\frac{\Delta f(t)}{\Delta t} = \frac{f(3+h)-f(3)}{3+h-3} = \frac{4(3+h)^2-36}{h} = \frac{4(h^2+6h+9)-36}{h} = \frac{4h^2+24h+36-36}{h}$$

$$= \frac{4h^2+24h}{h} = 4h+24$$

As h gets closer to 0, speed of object gets closer to 24 ft/s. So, we would say that 24 is the limit of $4h+24$ as h approaches 0. We express this notation as

$$\lim_{h \to 0} 4h+24 = 24$$

- A number L is the limit of a function $f(x)$ as x approaches some number c

$$\lim_{x \to c} f(x) = L$$

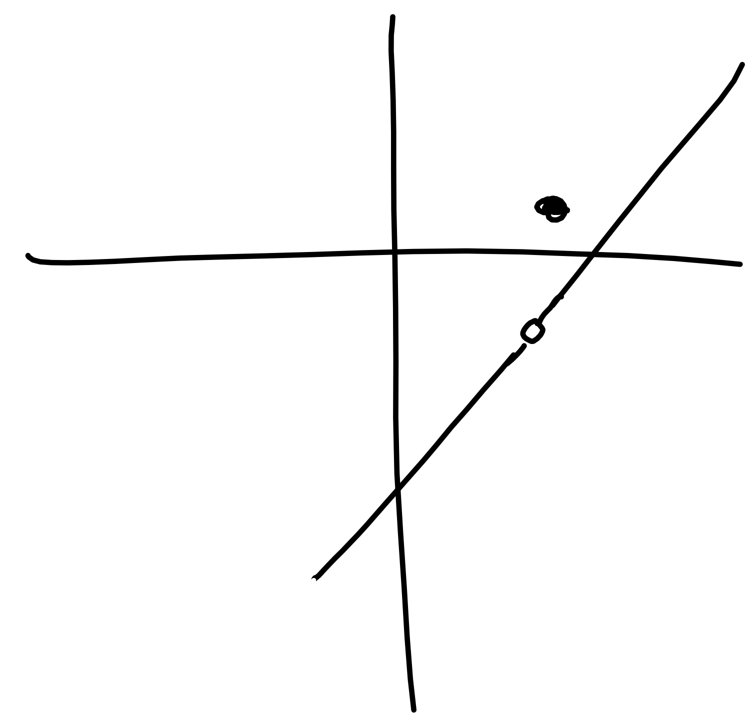
- Generally, to evaluate a limit of a function, we substitute c in for x

EX → $\lim_{x \to c} 2x^2 + 7x + 9 = 2c^2 + 7c + 9$

EX → $\lim_{x \to -2} 7x - 4 = 7(-2) - 4 = -18$

EX → $\lim_{x \to 4} \begin{cases} x - 7, & x \neq 4 \\ 1, & x = 4 \end{cases} = -3$

→ IGNORE!



- Limits do <u>NOT</u> exist under 3 conditions

1) $f(x)$ does not approach same L from both sides as x approaches c

$$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$$

<u>EX</u> → $\lim_{x \to 0} \frac{|x|}{x}$ DNE

2) $f(x)$ approaches ∞ or $-\infty$ as x approaches c

<u>EX</u> → $\lim_{x \to 0} \frac{1}{x^3}$ DNE

3) $f(x)$ oscillates between 2 fixed values as x approaches c

HW: p. 55 → 9-20