- Basic Properties of Limits (Let b + c be real #'s, n be pos. integer)

1) $\lim_{x \to c} b = b$

2) $\lim_{x \to c} x = c$

3) $\lim_{x \to c} x^n = c^n$

If $\lim_{x \to c} f(x) = L$ + $\lim_{x \to c} g(x) = K$

1) $\lim_{x \to c} [b \cdot f(x)] = b \cdot L$ (Scalar Multiple)

2) $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$ (Sum/Difference)

3) $\lim_{x \to c} [f(x) \cdot g(x)] = L \cdot K$ (Product)

4) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, $K \neq 0$ (Quotient)

5) $\lim_{x \to c} [f(x)]^n = L^n$ (Power)

EX → p.67, #40

$$\lim_{x\to c} f(x) = 27$$

A) $\lim_{x\to c} \sqrt[3]{f(x)} = \sqrt[3]{27} = 3$

B) $\lim_{x\to c} \frac{f(x)}{18} = \frac{27}{18} = \frac{3}{2}$

C) $\lim_{x\to c} [f(x)]^2 = 27^2 = 729$

D) $\lim_{x\to c} [f(x)]^{2/3} = 27^{2/3} = 9$

- Limit of Composite Functions

→ If $\lim_{x\to c} g(x) = L$ & $\lim_{x\to L} f(x) = f(L)$, then $\lim_{x\to c} f(g(x)) = f\left(\lim_{x\to c} g(x)\right) = f(L)$

$$\lim_{x\to c} f(g(x)) = \quad f(g(c)) = f(L)$$

- On occasion, substitution can produce what is called an <u>indeterminate form</u> $\left(\frac{0}{0}\right)$

- Strategies for Finding Limits
  - → Dividing Out

<u>EX 1</u> → $$\lim_{x\to 4} \frac{x^2-7x+12}{x^2-16} = \frac{16-28+12}{16-16} = \frac{0}{0} \Rightarrow \lim_{x\to 4} \frac{\cancel{(x-4)}(x-3)}{(x+4)\cancel{(x-4)}} = \lim_{x\to 4} \frac{x-3}{x+4} = \boxed{\frac{1}{8}}$$

<u>EX 2</u> → $$\lim_{x\to -5} \frac{x^2+15x+50}{x^2-25} = \frac{25-75+50}{25-25} = \frac{0}{0} \Rightarrow \lim_{x\to -5} \frac{\cancel{(x+5)}(x+10)}{\cancel{(x+5)}(x-5)} = \lim_{x\to -5} \frac{x+10}{x-5} = \frac{-5}{-10} = \boxed{-\frac{1}{2}}$$

- Rationalizing

EX1 → $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \frac{1-1}{0} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x\to 0} \frac{x+1-1}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \boxed{\frac{1}{2}}$

EX2 → $\lim_{x\to 0} \frac{x}{\sqrt{x+2}-\sqrt{2}} = \frac{0}{\sqrt{2}-\sqrt{2}} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{x}{\sqrt{x+2}-\sqrt{2}} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \lim_{x\to 0} \frac{x(\sqrt{x+2}+\sqrt{2})}{x+2-2}$

$= \lim_{x\to 0} \frac{\cancel{x}(\sqrt{x+2}+\sqrt{2})}{\cancel{x}} = \lim_{x\to 0} \sqrt{x+2}+\sqrt{2} = \sqrt{2}+\sqrt{2} = 2\sqrt{2}$

- "Squeeze" or "Sandwich" Theorem

→ If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly c itself, and $\lim_{x\to c} h(x) = \lim_{x\to c} g(x) = L$, then $\lim_{x\to c} f(x) = L$

- Limits w/ Trig Functions

(*) $$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

EX → $$\lim_{x\to 0} \frac{x+\sin x}{x} = \lim_{x\to 0} \frac{x}{x} + \frac{\sin x}{x} = \lim_{x\to 0} 1 + \frac{\sin x}{x} = 1+1 = 2$$

EX → $$\lim_{x\to 0} \frac{\sin 3x}{4x} \cdot \frac{\frac{3}{4}}{\frac{3}{4}} = \lim_{x\to 0} \frac{\frac{3}{4}\sin 3x}{3x} = \lim_{x\to 0} \frac{3}{4} \cdot \frac{\sin 3x}{3x} = \frac{3}{4} \cdot 1 = \frac{3}{4}$$

HW: p. 67 → 1-77 EOO