57

$$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{\frac{1}{3+x}\cdot\frac{3}{3}-\frac{1}{3}\cdot\frac{3+x}{3+x}}{x}$$

$$=\lim_{x\to 0}\frac{\frac{\cancel{3}-\cancel{3}-x}{3(3+x)}}{\frac{x}{1}}=\lim_{x\to 0}\frac{-\cancel{x}}{3\cancel{x}(3+x)}=\lim_{x\to 0}\frac{-1}{3(3+x)}=\boxed{\frac{-1}{9}}$$

61

$$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{(\cancel{x^2}+2x\Delta x+(\Delta x)^2)-\cancel{2x}-2(\Delta x)\cancel{+1}-\cancel{x^2}\cancel{+2x}\cancel{-1}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{2x(\cancel{\Delta x})+(\Delta x)^{\cancel{2}}-2(\cancel{\Delta x})}{\cancel{\Delta x}}=\lim_{\Delta x\to 0}2x+\Delta x-2=\boxed{2x-2}$$

- A function $f(x)$ is <u>continuous</u> at $x=c$ if the following conditions are met
  1) $f(c)$ is defined
  2) $\lim_{x \to c} f(x)$ exists
  3) $\lim_{x \to c} f(x) = f(c)$

- Discontinuities at $x=c$ can be classified as either <u>removable</u> or <u>non-removable</u>
  → To make removable, make $\lim_{x \to c} f(x) = f(c)$

<u>EX</u> → Describe continuities of following

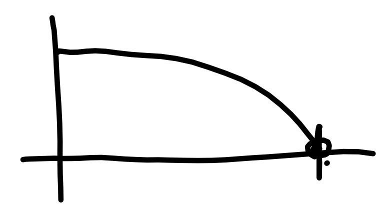
1) $f(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x+2)(x-2)}{x-2}$
   → continuous @ all x, $x \neq 2$ (non-continuous)
   → discontinuity is removable if $f(2)=4$

2) $f(x) = \dfrac{1}{x^2}$
   → non-continuous b/c $x \neq 0$
   → discontinuity is non-removable b/c $\lim_{x \to 0} \dfrac{1}{x^2} = \infty$

- Limits can be described as one-sided
  - If approach from <u>right</u> side $(x>c)$, then $\lim\limits_{x\to c^+} f(x) = L$
  - If approach from <u>left</u> side $(x<c)$, then $\lim\limits_{x\to c^-} f(x) = L$

(*) - Limit exists if $\lim\limits_{x\to c^+} f(x) = \lim\limits_{x\to c^-} f(x)$

<u>EX1</u> → $\lim\limits_{x\to 3^-} \sqrt{9-x^2} = 0$



$\lceil x \rceil$

<u>EX2</u> → $\lim\limits_{x\to 2^+} x^2+4 = 8$

→ $[[x]]$ = greatest integer n such that $n \le x$ (step functions) (floor function $\lfloor x \rfloor$)

<u>EX1</u> → $[[3.14]] = 3$

<u>EX2</u> → $[[-7.21]] = -8$

HW: p. 67 → 8-72 mult. 8

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$$\begin{array}{r} x^2-x+1 \\ x+1 \overline{\big)\, x^3+0x^2+0x+1} \\ -(x^3+x^2) \\ \hline -x^2+0x \\ -(-x^2-x) \\ \hline x+1 \end{array}$$

$$\lim_{x\to -1}\frac{x^3+1}{x+1}=\lim_{x\to -1}\frac{(x^2-x+1)\cancel{(x+1)}}{\cancel{x+1}}=\lim_{x\to -1}x^2-x+1$$

$$=1+1+1=3$$

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$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}=\lim_{x\to 3}\frac{\cancel{(x+1)-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)}=\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}=\frac{1}{2+2}=\frac{1}{4}$$

64

$$\lim_{x\to 16}\frac{4-\sqrt{x}}{x-16}\cdot\frac{4+\sqrt{x}}{4+\sqrt{x}}=\lim_{x\to 16}\frac{16-x}{(x-16)(4+\sqrt{x})}=\lim_{x\to 16}\frac{-1\cancel{(x-16)}}{\cancel{(x-16)}(4+\sqrt{x})}=\lim_{x\to 16}\frac{-1}{4+\sqrt{x}}=\frac{-1}{4+4}=-\frac{1}{8}$$

72

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=\lim_{x\to 0}\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{x}{1}}=\lim_{x\to 0}\frac{\sin^2 x}{x\cdot\cos^2 x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{\cos^2 x}=1\cdot\frac{0}{1}=0$$

-We have so far considered continuity on open intervals, generally $(-\infty, \infty)$. What about closed intervals? A function $f(x)$ is continuous on a closed interval $[a, b]$ if

1) continuous on (a, b)
2) $\lim_{x \to a^+} f(x) = f(a)$
3) $\lim_{x \to b^-} f(x) = f(b)$

EX → Describe continuity

$$f(x) = \begin{cases} 3 - x, & x \leq 0 \\ 3 + \frac{1}{2}x, & x > 0 \end{cases}, \quad [-1, 4]$$

$\lim_{x \to 0^-} f(x) = 3$

$\lim_{x \to 0^+} f(x) = 3$

Defined on $[-1, 4]$

- Properties of Continuities
  - If b is a real #, $f(x)$ & $g(x)$ are continuous at $x=c$, then the following functions are also continuous at $x=c$
    1) Scalar multiples $\Rightarrow$ $b \cdot f(x)$
    2) Sum/Difference $\Rightarrow$ $f(x) \pm g(x)$
    3) Product $\Rightarrow$ $f(x) \cdot g(x)$
    4) Quotient $\Rightarrow$ $\dfrac{f(x)}{g(x)}$, $g(c) \neq 0$
- The following types of functions are continuous at every point in their domains as well:
  - Polynomials
  - Rational Functions
  - Radical Functions
  - Basic Trig Functions (sin, cos, etc.)

- If a function $g(x)$ is continuous at $x=c$, & function $f(x)$ is continuous at $g(c)$, then the composite function $f(g(x))$ is continuous at $x=c$

EX → $g(x) = 4x+7$, continuous on $(-\infty, \infty)$
$f(x) = \sqrt{x}$, continuous on $[0, \infty)$
$\Big\}$ $f(g(x)) = \sqrt{4x+7}$

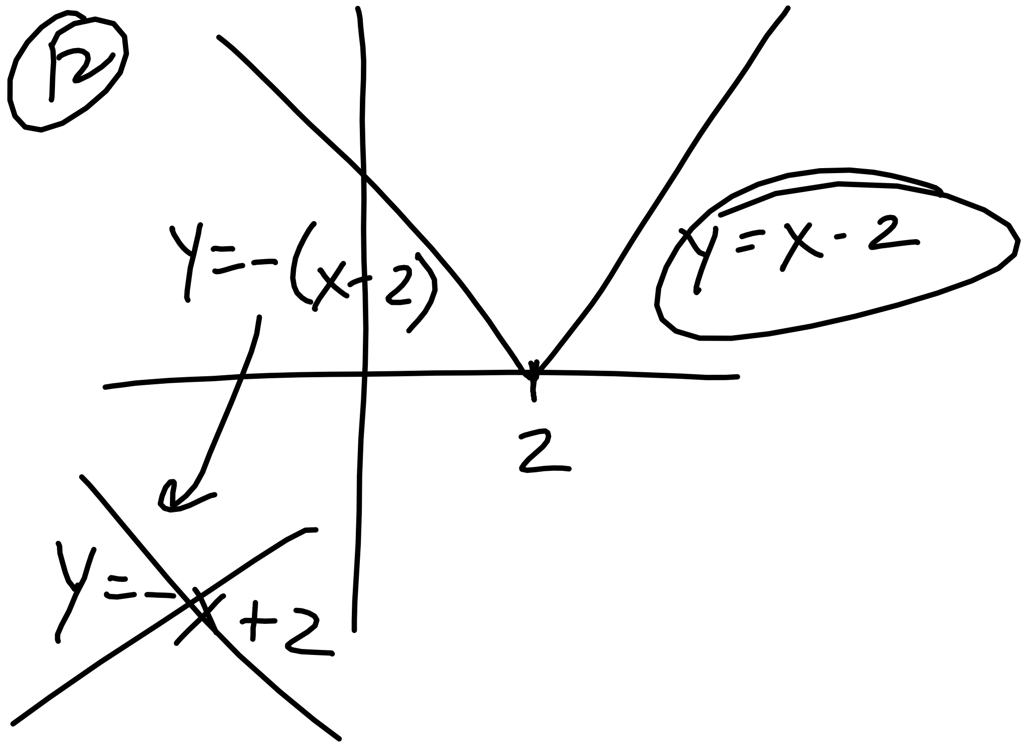
- Intermediate Value Theorem

→ If a function $f(x)$ is continuous on a closed interval $[a,b]$, and k is any number between $f(a)$ & $f(b)$, then there exists at least one number c in $[a,b]$ such that $f(c)=k$.

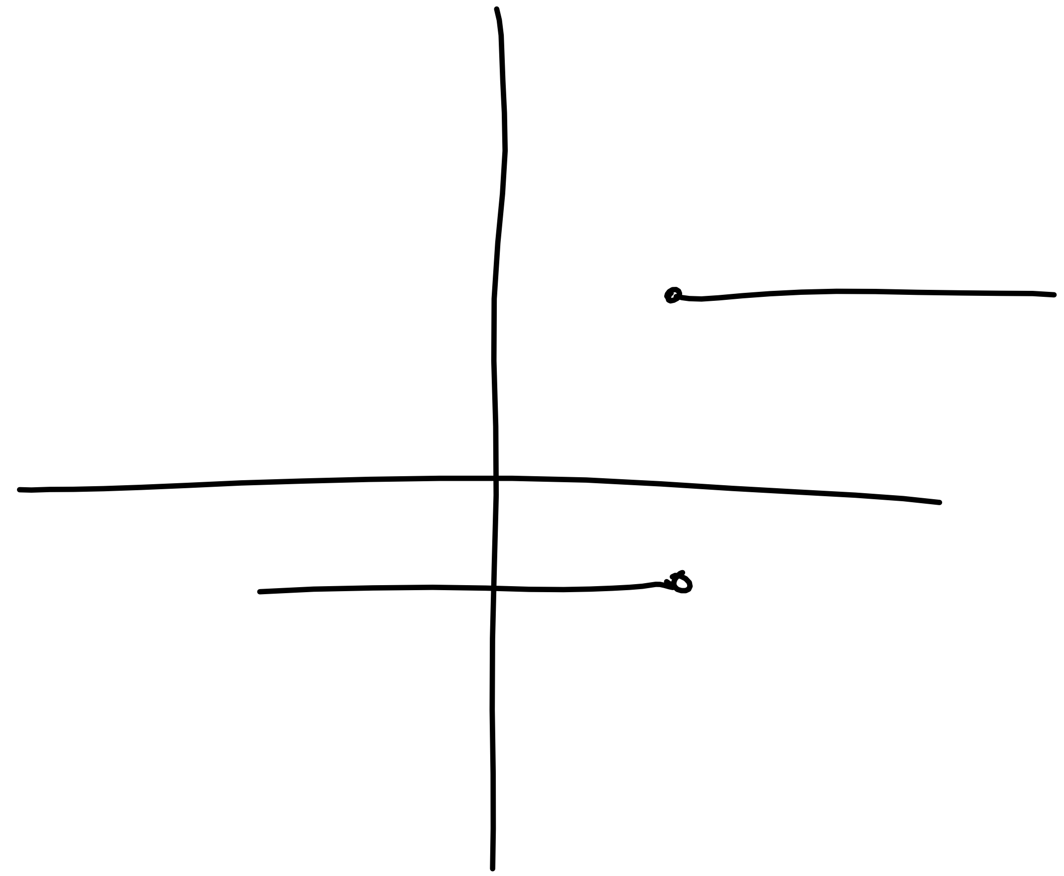
EX → $x = x^3 - 1 \Rightarrow x^3 - x - 1 = 0$ b/w $[1,2]$

HW: p. 78 → 2-40 even

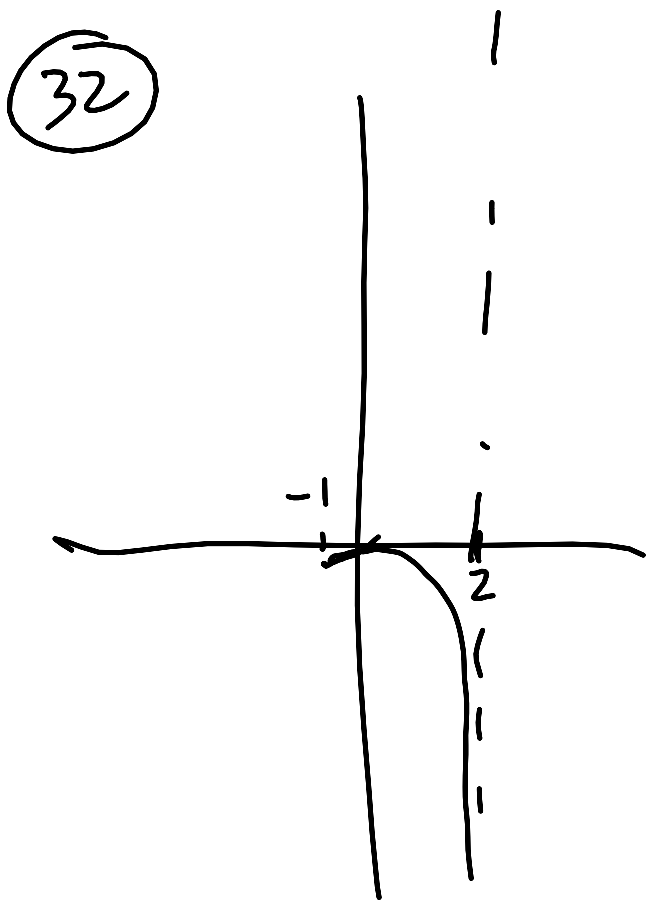
12



$$f(x) = \frac{|x-2|}{x-2} = \frac{x-2}{x-2} = 1$$



32



14 $\lim_{\Delta x \to 0^+} \frac{(x+\Delta x)^2 + x + \Delta x - x^2 - x}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{\cancel{x^2} + 2x(\Delta x) + (\Delta x)^2 + \cancel{x} + \Delta x - \cancel{x^2} - \cancel{x}}{\Delta x}$

$= \lim_{\Delta x \to 0^+} \frac{2x\cancel{(\Delta x)} + (\Delta x)^{\cancel{2}} + \cancel{\Delta x}^{1}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0^+} 2x + \Delta x + 1 = 2x+1$

HW: p. 79 → 7-27 EOO, 33, 37, 42-44, 46, 48, 75-78, 83-86, 100