- Another common method to approximate the area under a curve is the trapezoidal rule, which uses trapezoids. To approximate,

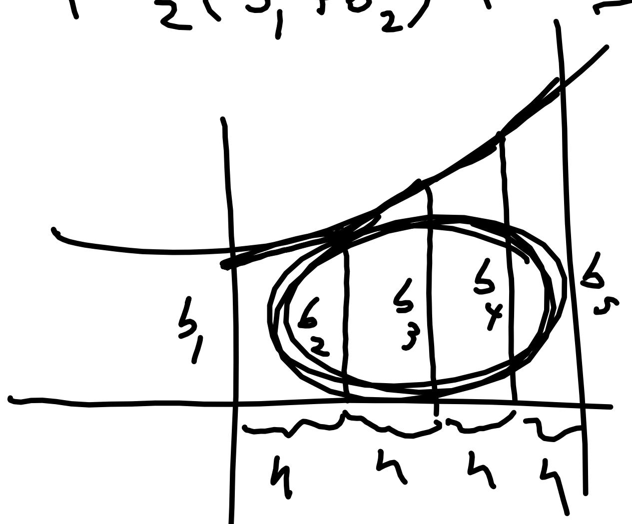
$$T_n = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$$

where $[a, b]$ is partitioned into n subintervals of equal length $h = \frac{b-a}{n}$.

Equivalently, $T = \frac{LRAM_n + RRAM_n}{2}$, where $LRAM_n$ & $RRAM_n$ are the Riemann sums using left & right endpoints, respectively.

$$A = \tfrac{1}{2}(b_1 + b_2)h \Rightarrow \tfrac{1}{2}h\left((b_1+b_2) + (b_2+b_3) + (b_3+b_4) + (b_4+b_5)\right)$$

$$= \tfrac{1}{2}h\left(b_1 + \underbrace{2b_2 + 2b_3 + 2b_4}_{} + b_5\right)$$



EX1 → $f(x) = \frac{1}{x}$, $[1,2]$, $n=5$

$h = \frac{2-1}{5} = 0.2$

$T_5 = \frac{0.2}{2}\left[f(1) + 2f(1.2) + 2f(1.4) + 2f(1.6) + 2f(1.8) + f(2)\right]$

$= 0.1\left[1 + 1.667 + 1.429 + 1.250 + 1.111 + 0.500\right] = \boxed{0.696}$

EX2 → $f(x) = x^2$, $[1,2]$, $n=4$

$h = \frac{2-1}{4} = 0.25$

$T_4 = \frac{0.25}{2}\left[f(1) + 2f(1.25) + 2f(1.5) + 2f(1.75) + f(2)\right]$

$= 0.125\left[1 + 3.125 + 4.5 + 6.125 + 4\right] = \boxed{2.344}$

HW: p. 314 → 1–11 odd

-A third method that is used to primarily approximate quadratics & other similar curves is Simpson's Rule. To approximate,

$$S_n = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

where n is an <u>even</u> number of equal subintervals

<u>EX1</u> → Use Simpson's Rule to approximate $f(x) = x^2$, $[1,2]$, $n=4$

$$S_4 = \frac{2-1}{3(4)}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right]$$

$$= \frac{1}{12}\left[1 + 6.25 + 4.5 + 12.25 + 4\right] = \frac{7}{3} = 2.333$$

EX2 → Use Simpson's Rule to approximate $f(x) = x \cdot \tan x$, $\left[0, \frac{\pi}{4}\right]$, $n=4$

$$S_4 = \frac{\frac{\pi}{4} - 0}{3(4)} \left[f(0) + 4f\left(\frac{\pi}{16}\right) + 2f\left(\frac{\pi}{8}\right) + 4f\left(\frac{3\pi}{16}\right) + f\left(\frac{\pi}{4}\right) \right]$$

$$= \frac{\pi}{48} \left[0 + 0.1562 + 0.3253 + 1.5744 + 0.7854 \right] = \boxed{0.186}$$

<u>HW</u>: p. 314 → 2 - 18 even