- Sigma Notation

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

n ← ending point

$k=1$ ← starting point

- In evaluating definite integrals, we are interested in Riemann sums (examples of which are LRAM, RRAM, MRAM) b/c they use partitions of the interval to help calculate the area under the curve. To do so,
  - → Select n subintervals
  - → Use ΔX_k as width of subinterval (width of rectangle)
  - → In the subinterval, pick some number c_k, & evaluate $f(c_k)$ to get height of the rectangle.
  - → Sum of the products → $S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta X_k$

→ Definition: Let f be a function defined on a closed interval $[a,b]$. For any interval P of $[a,b]$, let the numbers c_k be chosen arbitrarily in the subintervals $[x_{k-1}, x_k]$. If there exists a number I such that

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k = I$$

no matter how P + the c_k's are chosen, then f is integratable on $[a,b]$ + I is the definite integral of f over $[a,b]$

NOTE: While Riemann sums tend to the same limit in which the norm $\|P\| \to 0$, we only consider the limit of regular partitions (those of the same length)

→ <u>Definition</u>: Let f be continuous on $[a,b]$ & let $[a,b]$ be partitioned into n subintervals of equal length $\Delta x = \frac{b-a}{n}$. Then the definite integral of f over $[a,b]$ is given by

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\cdot\Delta x$$

where each c_k is chosen arbitrarily in the kth subinterval

- Changing this definition to Leibniz notation

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\cdot\Delta x = \int_a^b \underbrace{f(x)}_{\text{integrand}}\,dx$$

upper limit of integration

integrand

lower limit of integration

→ Definition: If $y=f(x)$ is non-negative & integratable over a closed interval $[a,b]$, then the area under the curve $y=f(x)$ from a to b is the integral of f from a to b,

$$A=\int_a^b f(x)\,dx$$

- Properties of Definite Integrals

1) Zero: $\int_a^a f(x)\,dx=0$
2) Additivity: $\int_a^b f(x)\,dx+\int_b^c f(x)\,dx=\int_a^c f(x)\,dx$
3) Order of Integration: $\int_b^a f(x)\,dx=-\int_a^b f(x)\,dx$
4) Constant Multiple: $\int_a^b k\cdot f(x)\,dx=k\cdot\int_a^b f(x)\,dx$
5) Sum/Difference: $\int_a^b (f(x)\pm g(x))\,dx=\int_a^b f(x)\,dx\pm\int_a^b g(x)\,dx$

HW: p. 278 → 13-22, 33-45 odd

HW: p. 279 → 34 – 46 even, 47, 48