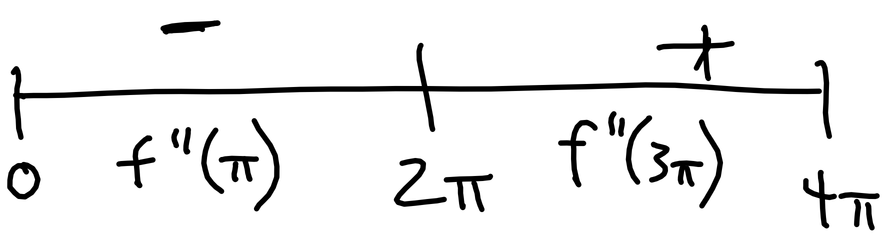
(21) $f(x) = \sin\frac{x}{2}$, $[0, 4\pi]$

$f'(x) = \cos\frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2}\cos\frac{x}{2}$

$f''(x) = -\frac{1}{2}\sin\frac{x}{2} \cdot \frac{1}{2} = -\frac{1}{4}\sin\frac{x}{2} = 0$

$x = 0, 2\pi, 4\pi$



$f(x)$ is concave down on $[0, 2\pi)$

concave up on $(2\pi, 4\pi]$

(23) $f(x) = \sec\left(x - \frac{\pi}{2}\right)$, $(0, 4\pi)$

$f'(x) = \sec\left(x - \frac{\pi}{2}\right)\tan\left(x - \frac{\pi}{2}\right)$

$f''(x) = \sec\left(x - \frac{\pi}{2}\right)\sec^2\left(x - \frac{\pi}{2}\right) + \tan\left(x - \frac{\pi}{2}\right)\sec\left(x - \frac{\pi}{2}\right)\tan\left(x - \frac{\pi}{2}\right) = 0$

$\sec^3\left(x - \frac{\pi}{2}\right) + \sec\left(x - \frac{\pi}{2}\right)\tan^2\left(x - \frac{\pi}{2}\right) = 0$

$\sec^3\left(x - \frac{\pi}{2}\right) = -\sec\left(x - \frac{\pi}{2}\right)\tan^2\left(x - \frac{\pi}{2}\right)$

$\sec^2\left(x - \frac{\pi}{2}\right) = -\tan^2\left(x - \frac{\pi}{2}\right)$

$\frac{1}{\cos^2\left(x - \frac{\pi}{2}\right)} = \frac{-\sin^2\left(x - \frac{\pi}{2}\right)}{\cos^2\left(x - \frac{\pi}{2}\right)}$

$\sin^2\left(x - \frac{\pi}{2}\right) = -1$

(25) $f(x) = 2\sin x + \sin 2x, \ [0, 2\pi]$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x = 0$

$-2\sin x = 4\sin 2x$

$-\frac{1}{2}\sin x = \sin 2x$

$-$ $\quad +$ $\quad -$ $\quad +$

0 $\quad f''(\frac{\pi}{3})$ $\quad 1.823$ $\quad f''(\frac{3\pi}{4})$ $\quad \pi$ $\quad f''(4)$ $\quad 4.459$ $\quad f''(5)$ $\quad 2\pi$

$f(x)$ is concave up on $(1.823, \pi), (4.459, 2\pi]$

concave down on $[0, 1.823), (\pi, 4.459)$

(11) $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12 = 0$

$x = 2$

$-$ $\quad +$

$f''(1)$ $\quad 2$ $\quad f''(3)$

$f(x)$ is Concave up $(2, \infty)$

$f(x)$ is Concave down $(-\infty, 2)$

(*) 2nd Derivative Test

↳ Let $f'(c)=0$ & the 2nd derivative exist on f. If $f''(c)>0$, then $f(x)$ has a relative minimum @ $(c, f(c))$; if $f''(c)<0$, then $f(x)$ has a relative maximum @ $(c, f(c))$

<u>NOTE</u>: If $f''(c)=0$, then test fails

<u>EX1</u> → $f(x)=x^2+3x-8$

$f'(x)=2x+3=0$

$x=-\frac{3}{2}$

$f''(x)=2 \rightarrow f(x)$ has rel. min. @ $x=-\frac{3}{2}$

<u>EX2</u> → $f(x)=x^3-9x^2+27x$

$f'(x)=3x^2-18x+27=3(x^2-6x+9)=3(x-3)^2=0$

$x=3$

$f''(x)=6x-18$

$f''(3)=6(3)-18=0 \rightarrow$ TEST FAILS

HW: p. 195 → 27-39 odd