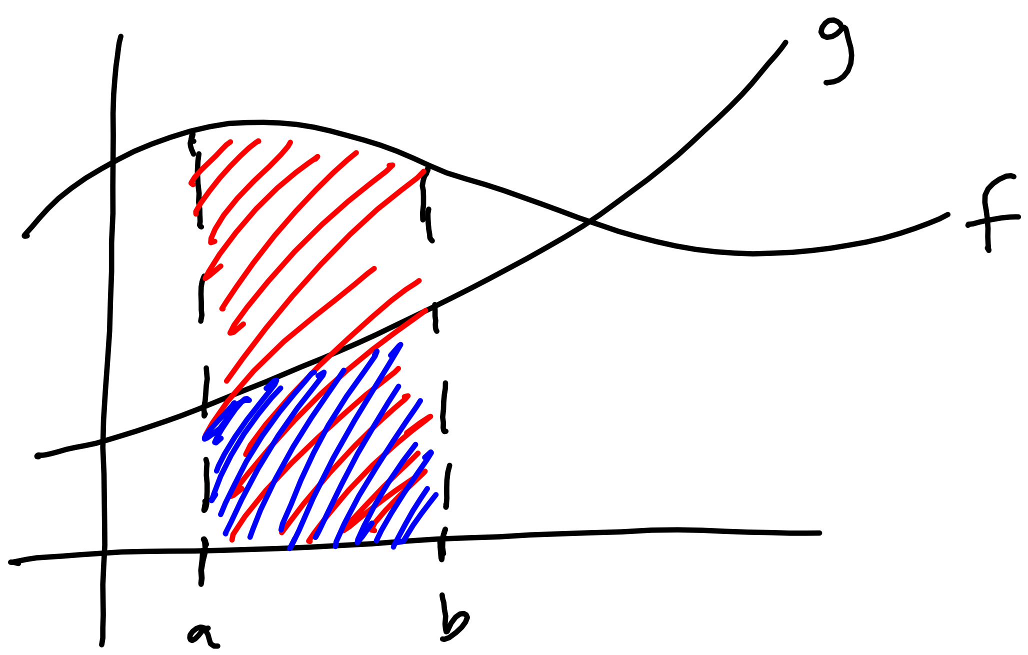
Applications of Integration



How can we find the area between f + g?

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \big(f(x) - g(x)\big)\,dx$$

EX1 → Find area b/w $y = \sec^2 x$ + $y = \sin x$ from $x = 0$ to $\cancel{x} = \frac{\pi}{4}$

$$\int_0^{\pi/4} \sec^2 x - \sin x\,dx = \tan x + \cos x \Big|_0^{\pi/4} = \left[\tan\left(\tfrac{\pi}{4}\right) + \cos\left(\tfrac{\pi}{4}\right)\right] - \left[\tan(0) + \cos(0)\right]$$

$$= 1 + \frac{\sqrt{2}}{2} - 0 - 1 = \boxed{\frac{\sqrt{2}}{2}}$$

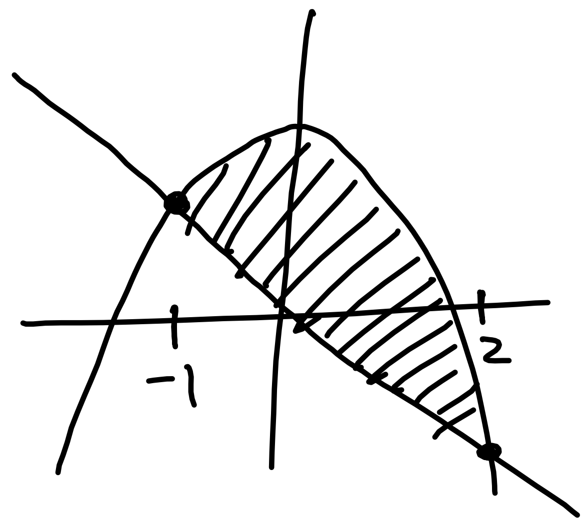
EX2 → Find area enclosed by $y = 2 - x^2$ & $y = -x$

$2 - x^2 = -x$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$



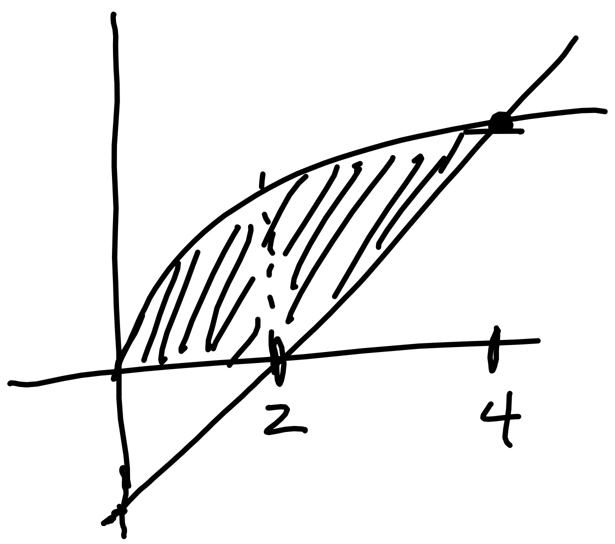
$$\int_{-1}^{2} 2 - x^2 - (-x)\, dx = \int_{-1}^{2} 2 - x^2 + x\, dx$$

$$= 2x - \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= \left[2(2) - \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2\right] - \left[2(-1) - \frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2\right]$$

$$= \left[4 - \frac{8}{3} + 2\right] - \left[-2 + \frac{1}{3} + \frac{1}{2}\right] = \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

EX3 → Find the area bounded above by $y=\sqrt{x}$ & below by x-axis & $y=x-2$



$$\int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x}-(x-2)\,dx$$

$$\left[\frac{2}{3}x^{3/2}\Big|_0^2\right] + \left[\frac{2}{3}x^{3/2}-\frac{1}{2}x^2+2x\Big|_2^4\right]$$

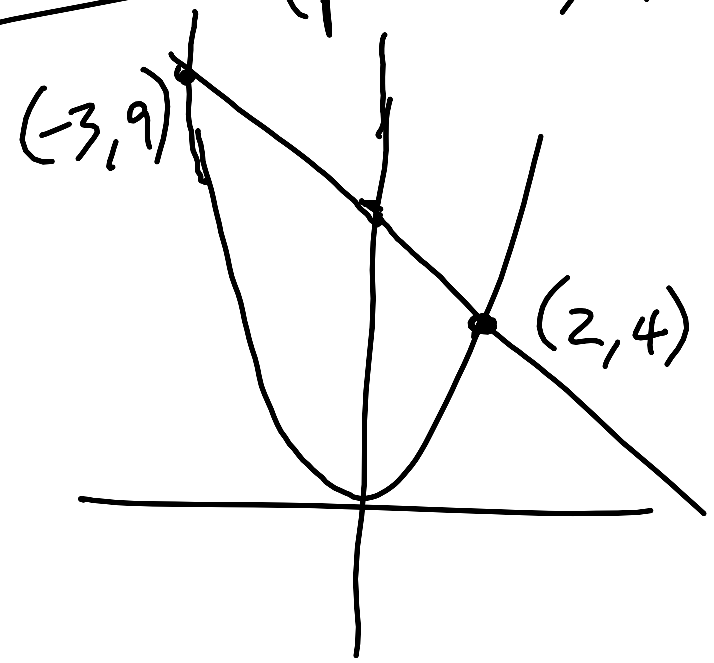
$$= \left[\frac{2}{3}(2)^{3/2}-0\right]+\left[\frac{2}{3}(4)^{3/2}-\frac{1}{2}(4)^2+2(4)\right]-\left[\frac{2}{3}(2)^{3/2}-\frac{1}{2}(2)^2+2(2)\right]$$

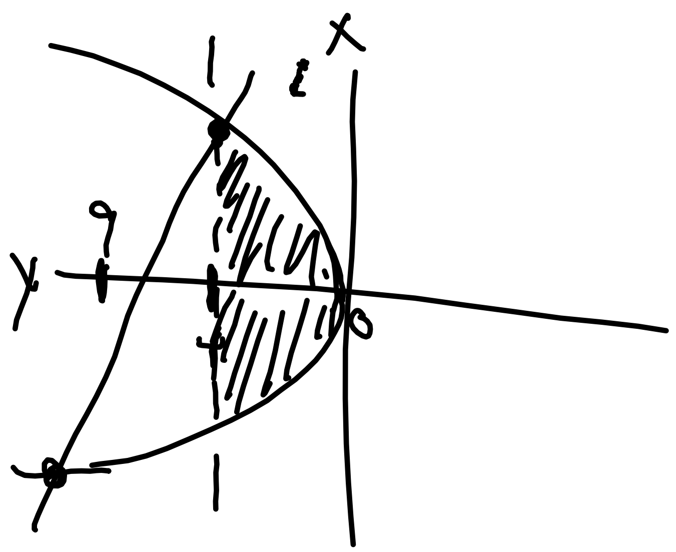
$$= \cancel{\frac{4\sqrt{2}}{3}}+\frac{16}{3}-\cancel{8+8}-\left(\cancel{\frac{4\sqrt{2}}{3}}-2+4\right)=\frac{16}{3}-2=\boxed{\frac{10}{3}}$$

HW: p. 452 → 1-6, 17-31 odd, 43-47 odd

EX4 → (p. 452, #14) Find area between $x=\pm\sqrt{y}$ & $x=6-y$

$\downarrow$ $y=x^2$ $\qquad$ $\downarrow$ $y=6-x$





$$2\int_0^4 \sqrt{y}\,dy + \int_4^9 6-y-(-\sqrt{y})\,dy$$

$$=2\left[\frac{2}{3}y^{3/2}\Big|_0^4\right] + \left[6y-\frac{1}{2}y^2+\frac{2}{3}y^{3/2}\Big|_4^9\right]$$

$$=2\left[\frac{2}{3}(4)^{3/2}-0\right]+\left[6(9)-\frac{1}{2}(9)^2+\frac{2}{3}(9)^{3/2}\right]-\left[6(4)-\frac{1}{2}(4)^2+\frac{2}{3}(4)^{3/2}\right]$$

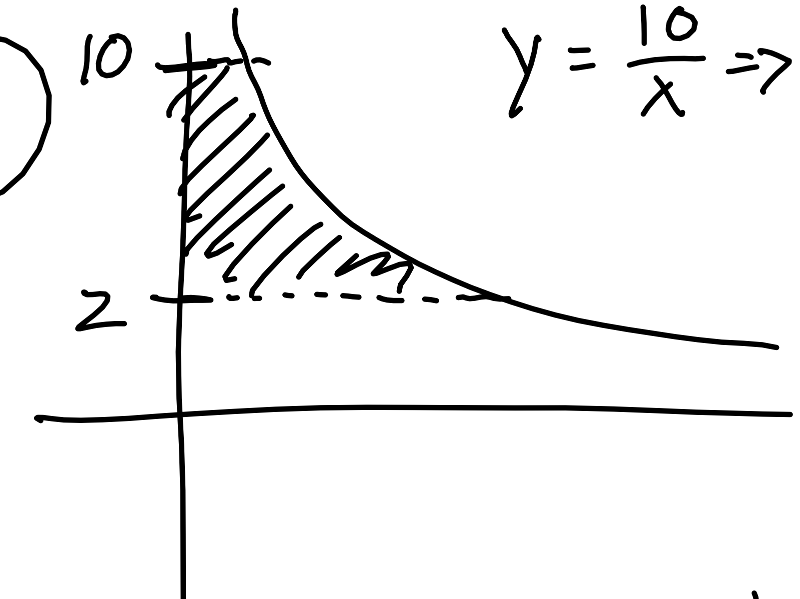
$$=\frac{32}{3}+54-\frac{81}{2}+18-24+8-\frac{16}{3}$$

$$=\frac{16}{3}+56-\frac{81}{2}=\frac{16}{3}+\frac{31}{2}=\frac{32}{6}+\frac{93}{6}=\boxed{\frac{125}{6}}$$

(5) $\int_{-1}^{0} 3(x^3-x)\,dx + \int_{0}^{1} 0-3(x^3-x)\,dx$

(6) $\int_{0}^{1} (x-1)-(x-1)^3\,dx + \int_{1}^{2} (x-1)-(x-1)^3\,dx$

(31)



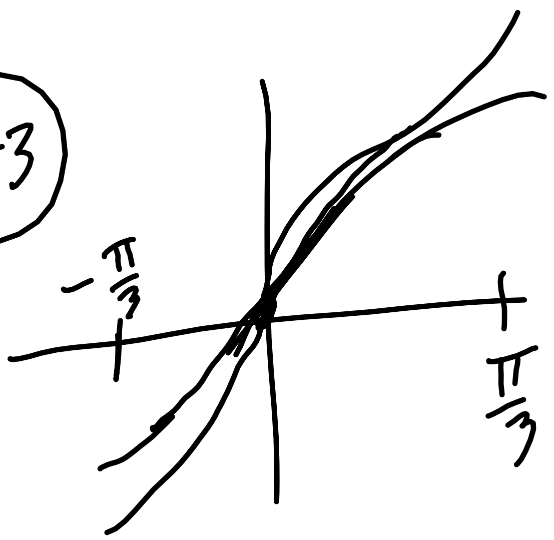
$y=\frac{10}{x} \Rightarrow x=\frac{10}{y}$

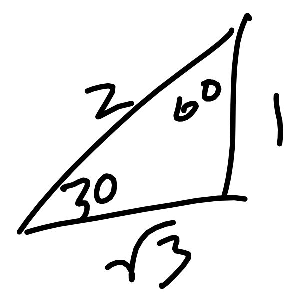
$\int_{2}^{10} \frac{10}{y}\,dy = 10\cdot\ln|y|\Big|_{2}^{10}$

$= 10\cdot\ln 10 - 10\ln 2 = 10(\ln 10-\ln 2) = 10\ln\left(\frac{10}{2}\right)$

$= 10\ln 5$

(43)





$2\int_{0}^{\frac{\pi}{3}} \sin x-\tan x\,dx$

$= 2(-\cos x)-2\ln|\tan x+\sec x|\Big|_{0}^{\frac{\pi}{3}}$

$= 2\left(-\cos\left(\frac{\pi}{3}\right)\right)-2\ln\left|\tan\left(\frac{\pi}{3}\right)+\sec\left(\frac{\pi}{3}\right)\right| - 2\cos(0)+2\ln|\sec(0)+\tan(0)|$

$= -1-2\ln|\sqrt{3}+2|-2+\cancel{2\ln|1+0|}^{\,0} = -1-2\ln|\sqrt{3}+2|$

HW: p. 452 → 13, 14, 18 - 32 even, 44 - 48 even,
72, 73