(22) $f(x) = x^3 - 6x^2 + 15$

$$f'(x) = 3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0, 4$$

$+$ | $-$ | $+$

$f'(-1)$ $\quad 0 \quad$ $f'(1)$ $\quad 4 \quad$ $f'(5)$

$f(x)$ is inc on $(-\infty, 0), (4, \infty)$

dec on $(0, 4)$

rel max @ $x = 0$

rel min @ $x = 4$

(32) $f(x) = |x+3| - 1$

$-$ | $+$

-3

$f(x)$ is dec on $(-\infty, -3)$

inc on $(-3, \infty)$

(14) $f(x) = x + \frac{4}{x} \rightarrow 4x^{-1}$

$$f'(x) = 1 - 4x^{-2} = 0 \qquad 1 - \frac{4}{x^2}$$

$$1 = \frac{4}{x^2}$$

$$x^2 = 4 \Rightarrow x = \pm 2, \text{ V.A. @ } x = 0$$

$+$ | $-$ | $-$ | $+$

$f'(-3)$ $\quad -2 \quad$ $f'(-1)$ $\quad 0 \quad$ $f'(1)$ $\quad 2 \quad$ $f'(3)$

$f(x)$ inc on $(-\infty, -2)$, $(2, \infty)$

dec on $(-2, 0), (0, 2)$

(40) $f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + (\cos x)(\cos x)$

$= \cos^2 x - \sin^2 x = 0$

$\sin^2 x = \cos^2 x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$+ \quad - \quad -$

$0 \quad f'(\frac{\pi}{6}) \quad \frac{\pi}{4} \quad f'(\pi) \quad \frac{5\pi}{4} \quad f'(\frac{3\pi}{2}) \quad 2\pi$

$\frac{3}{4} - \frac{1}{4} \qquad 0 - 1 \qquad 0 - 1$

$f(x)$ inc on $[0, \frac{\pi}{4})$

dec on $(\frac{\pi}{4}, 2\pi]$

(46) $f(x) = \frac{\sin x}{1 + \cos^2 x}$

$f'(x) = \frac{(1+\cos^2 x)(\cos x) - (\sin x)(-2\cos x \sin x)}{(1+\cos^2 x)^2}$

$= \frac{\cos x + \cos^3 x + 2\cos x \sin^2 x}{(1+\cos^2 x)^2}$

$\sin^2 x + \sin^2 x$

$= \frac{\cos x\left(1 + \cos^2 + \cancel{2\sin^2 x}\right)}{(1+\cos^2 x)^2}$

$= \frac{\cos x\left(1 + 1 + \sin^2 x\right)}{(1+\cos^2 x)^2} = \frac{(\cos x)(2 + \sin^2 x)}{(1+\cos^2 x)^2}$

- concavity → curving upward or downward

→ Let f be differentiable over an open interval. If f' is increasing, then f is concave upward; if f' is decreasing, then f is concave downward

- Since derivatives tell when a function is increasing or decreasing, we need to take the derivative of the 1st derivative, i.e., the 2nd derivative

→ To test for concavity, let $f''(x)$ exist on an interval

→ $f''(x) > 0$ → concave up

→ $f''(x) < 0$ → concave down

- Test is same as using 1st derivative to see where $f(x)$ is increasing or decreasing

EX → $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0, 2$

+		−		+
$f''(-1)$	0	$f''(1)$	2	$f''(3)$

$f(x)$ is

concave up on $(-\infty, 0), (2, \infty)$

concave down on $(0, 2)$

HW: p. 195 → 1-25 odd