(13) $\displaystyle\int \frac{x^2-3x+2}{x+1}\,dx$

$$\begin{array}{r|rrr} -1 & 1 & -3 & 2 \\ & & -1 & 4 \\ \hline & 1 & -4 & 6 \end{array}$$

$= \displaystyle\int x-4+\frac{6}{x+1}\,dx$

$u = x+1$

$du = dx$

$= \displaystyle\int x-4\,dx + \int \frac{6}{u}\,du$

$= \frac{1}{2}x^2 - 4x + 6\ln|u| + C$

$= \frac{1}{2}x^2 - 4x + 6\ln|x+1| + C$

(51) $\displaystyle\int_0^2 \frac{x^2-2}{x+1}\,dx = \int_0^2 x-1-\frac{1}{x+1}\,dx$

$$\begin{array}{r|rrr} -1 & 1 & 0 & -2 \\ & & -1 & 1 \\ \hline & 1 & -1 & -1 \end{array}$$

$= \frac{1}{2}x^2 - x - \ln|x+1| \Big|_0^2$

$= \left[\frac{1}{2}\cdot 4 - 2 - \ln|3|\right] - \left[0-0-\ln|1|\right]$

$= -\ln 3$

(27) $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

$u = \sqrt{x}-3$

$du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du = dx$

$2(u+3)\,du = dx$

$= \displaystyle\int \frac{u+3}{u}\cdot 2(u+3)\,du$

$= \displaystyle\int \frac{2(u+3)^2}{u}\,du = 2\int \frac{u^2+6u+9}{u}\,du$

$= 2\displaystyle\int u+6+\frac{9}{u}\,du = 2\left(\frac{1}{2}u^2+6u+9\ln|u| + C\right)$

$= u^2 + 12u + 18\ln|u| + C_1$

$= (\sqrt{x}-3)^2 + 12(\sqrt{x}-3) + 18\ln|\sqrt{x}-3| + C_1$

Exponential Functions

- inverse functions of logarithmic equations

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

EX1 → $\ln(e^{4x}) = 4x$

EX2 → $e^{\ln 7y} = 7y$

EX3 → $\ln(5x-4) = 7$

$$e^{\ln(5x-4)} = e^7$$

$$5x-4 = e^7$$

$$\boxed{x = \frac{e^7+4}{5}}$$

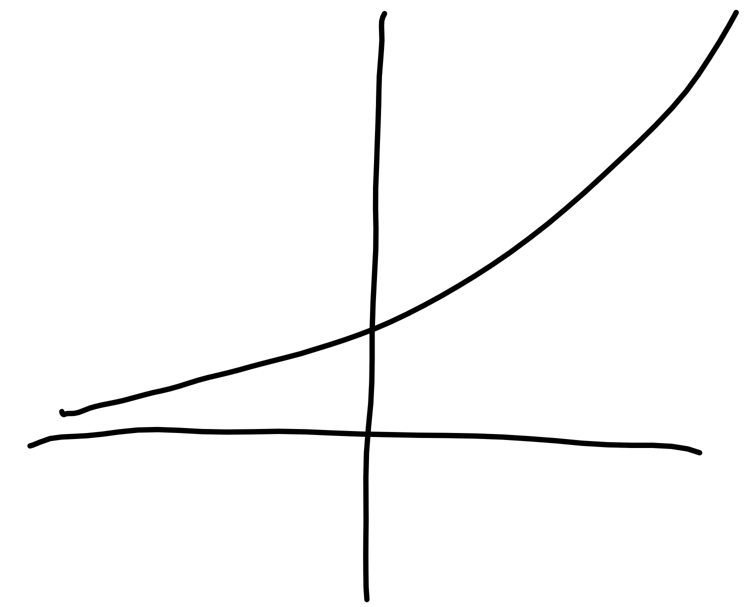
EX4 → $e^{3x-5} = 8$

$$\ln(e^{3x-5}) = \ln 8$$

$$3x-5 = \ln 8$$

$$\boxed{x = \frac{\ln 8 + 5}{3}}$$

- Properties
  - → Domain: $(-\infty, \infty)$
  - → Range: $(0, \infty)$
  - → continuously increasing
  - → concave upward
  - → 1-to-1
  - → $\lim_{x \to \infty} e^x = \infty$, $\lim_{x \to -\infty} e^x = 0$



- Derivative of Exponential Function
  - → $\frac{d}{dx}[e^x] = e^x$
  - → $\frac{d}{dx}[e^u] = u' e^u$

EX1 → $\frac{d}{dx}[e^{5x}] = 5e^{5x}$

EX2 → $\frac{d}{dx}[e^{\ln(x^2)}] = \frac{d}{dx}[x^2] = 2x$

EX3 → $\frac{d}{dx}[e^{\cos 3x}] = -\sin 3x \cdot 3 e^{\cos 3x} = -3\sin 3x \, e^{\cos 3x}$

HW: p. 356 → 21-24, 33-61 odd, 65

- Integrating Exponential Functions

$\rightarrow \int e^x\,dx = e^x + C$

$\rightarrow \int e^u\,du = e^u + C$

EX 1 $\rightarrow \int e^{5x+4}\,dx$

$u = 5x+4$

$du = 5dx \Rightarrow \frac{1}{5}du = dx$

$= \int e^u \cdot \frac{1}{5}du = \frac{1}{5}e^u + C = \boxed{\frac{1}{5}e^{5x+4} + C}$

EX 2 $\rightarrow \int 8x \cdot e^{4x^2}\,dx$

$u = 4x^2$

$du = 8x\,dx$

$= \int e^u\,du = e^u + C = \boxed{e^{4x^2} + C}$

EX 3 $\rightarrow \int_0^2 \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$

$du = e^x dx$

$= \int_{x=0}^{x=2} \frac{1}{u}\,du$

$= \ln|u| \Big|_{x=0}^{x=2}$

$= \ln|1+e^x| \Big|_0^2 = \ln(1+e^2) - \ln|1+e^0|$

$= \ln(1+e^2) - \ln 2 = \boxed{\ln\left(\frac{1+e^2}{2}\right)}$

HW: p. 356 → 34–62 even, 66–70

(47) $F(x)=\int_{\pi}^{\ln x} \cos e^{t}\,dt$

$F'(x)=\cos e^{\ln x}\cdot\frac{1}{x}$

$=\frac{\cos x}{x}$

(55) $f(x)=e^{-x}\ln x,\ (1,0)$

$f'(x)=e^{-x}\cdot\frac{1}{x}+\ln x\cdot -e^{-x}$

$f'(x)=\frac{1}{xe^{x}}-\frac{\ln x}{e^{x}}$

$f'(1)=\frac{1}{1\cdot e^{1}}-\cancel{\frac{\ln(1)}{e^{1}}}^{\,0}=\frac{1}{e}$

$\boxed{y-0=\frac{1}{e}(x-1)}$

(58) $e^{xy}+x^{2}-y^{2}=10$

$e^{xy}\left(x\cdot\frac{dy}{dx}+y\cdot 1\right)+2x-2y\frac{dy}{dx}=0$

$\underline{xe^{xy}\frac{dy}{dx}}+ye^{xy}+2x-\underline{2y\frac{dy}{dx}}=0$

$xe^{xy}\frac{dy}{dx}-2y\frac{dy}{dx}=-ye^{xy}-2x$

$\frac{dy}{dx}\left(xe^{xy}-2y\right)=-ye^{xy}-2x$

$\boxed{\frac{dy}{dx}=\frac{-ye^{xy}-2x}{xe^{xy}-2y}}$

60
61
68

(59) $xe^y + ye^x = 1,\ (0,1)$

$$x\cdot e^y\cdot\frac{dy}{dx} + e^y\cdot 1 + y\cdot e^x + e^x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(xe^y + e^x\right) = -e^y - ye^x$$

$$\frac{dy}{dx} = \frac{-e^y - ye^x}{xe^y + e^x} = \frac{-e^1 - 1\cdot e^0}{0\cdot e^1 + e^0}$$

$$= \frac{-e-1}{0+1} = -e-1$$

$$\boxed{y-1 = (-e-1)(x-0)}$$

(60) $1 + \ln xy = e^{x-y},\ (1,1)$

$$\frac{1}{xy}\cdot\left(x\frac{dy}{dx} + y\right) = e^{x-y}\cdot\left(1 - \frac{dy}{dx}\right)$$

$$\frac{1}{1\cdot 1}\cdot\left(1\frac{dy}{dx} + 1\right) = e^{1-1}\left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} + 1 = 1 - \frac{dy}{dx}$$

$$2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 0$$

$$\boxed{y-1 = 0(x-1)} \Rightarrow \underline{y=1}$$

(61)

$$f(x) = (3+2x)e^{-3x}$$

$$f'(x) = (3+2x)\cdot -3e^{-3x} + e^{-3x}\cdot 2 = (-9-6x)e^{-3x} + 2e^{-3x}$$

$$= e^{-3x}(-9-6x+2) = e^{-3x}(-7-6x)$$

$$f''(x) = (-7-6x)\cdot -3e^{-3x} + e^{-3x}\cdot -6$$

$$= (21+18x)e^{-3x} - 6e^{-3x} = (21+18x-6)e^{-3x}$$

$$\boxed{f''(x) = (15+18x)e^{-3x}}$$

68

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-3)^2}{2}}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-(x-3)^2}{2}} \cdot \frac{-\cancel{2}(x-3)}{\cancel{2}}$$

$$= \frac{-(x-3)}{\sqrt{2\pi}} \cdot e^{\frac{-(x-3)^2}{2}}$$

$+ \quad | \quad -$

$\quad\quad 3$

max @ $x = 3$

inc on $(-\infty, 3)$

dec on $(3, \infty)$

$$f''(x) = 0$$

$+ \quad | \quad - \quad | \quad +$

$\quad\quad 2 \quad\quad\quad 4$

concave up on $(-\infty, 2), (4, \infty)$

concave down on $(2, 4)$

POI @ $x = 2, 4$

HW: p. 358 → 85–106

(89) $\int \frac{e^{-x}}{1+e^{-x}}\,dx$

$u = 1 + e^{-x}$

$du = -e^{-x}\,dx$

$\Downarrow$

$-du = e^{-x}\,dx$

$= \int \frac{-1}{u}\,du$

$= -\ln|u| + C = -\ln|1+e^{-x}| + C$

(94) $\int \frac{2e^{x} - 2e^{-x}}{(e^{x}+e^{-x})^{2}}\,dx$

$u = e^{x} + e^{-x}$

$du = e^{x} - e^{-x}\,dx$

$\Downarrow$

$2du = 2e^{x} - 2e^{-x}\,dx$

$= \int \frac{2}{u^2}\,du$

$= -2u^{-1} + C$

$= \frac{-2}{e^{x}+e^{-x}} + C$

(96) $\int \frac{e^{2x} + 2e^{x} + 1}{e^{x}}\,dx$

$= \int \frac{e^{2x}}{e^{x}} + \frac{2e^{x}}{e^{x}} + \frac{1}{e^{x}}\,dx$

$= \int e^{x} + 2 + e^{-x}\,dx$

$= e^{x} + 2x - e^{-x} + C$

103

(98) $\int \ln(e^{2x-1})\,dx$

$= \int 2x - 1\,dx = x^2 - x + C$

97 $\int e^{-x}\tan(e^{-x})\,dx$

$u = e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$= \int -\tan u\,du$

$= -\ln|\cos u| + C = -\ln|\cos(e^{-x})| + C$

103 $\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}\,dx$

$u = \frac{3}{x}$

$du = -3x^{-2}\,dx = \frac{-3}{x^2}\,dx$

$-\frac{1}{3}\,du = \frac{1}{x^2}\,dx$

$= \int_1^3 e^{\frac{3}{x}} \cdot \frac{1}{x^2}\,dx$

$= \int_{x=1}^{x=3} -\frac{1}{3}e^u\,du$

$= -\frac{1}{3}e^u \Big|_{x=1}^{x=3} = -\frac{1}{3}e^{\frac{3}{x}}\Big|_1^3 = \boxed{-\frac{1}{3}e^1 + \frac{1}{3}e^3}$

$= \frac{e^3 - e}{3}$