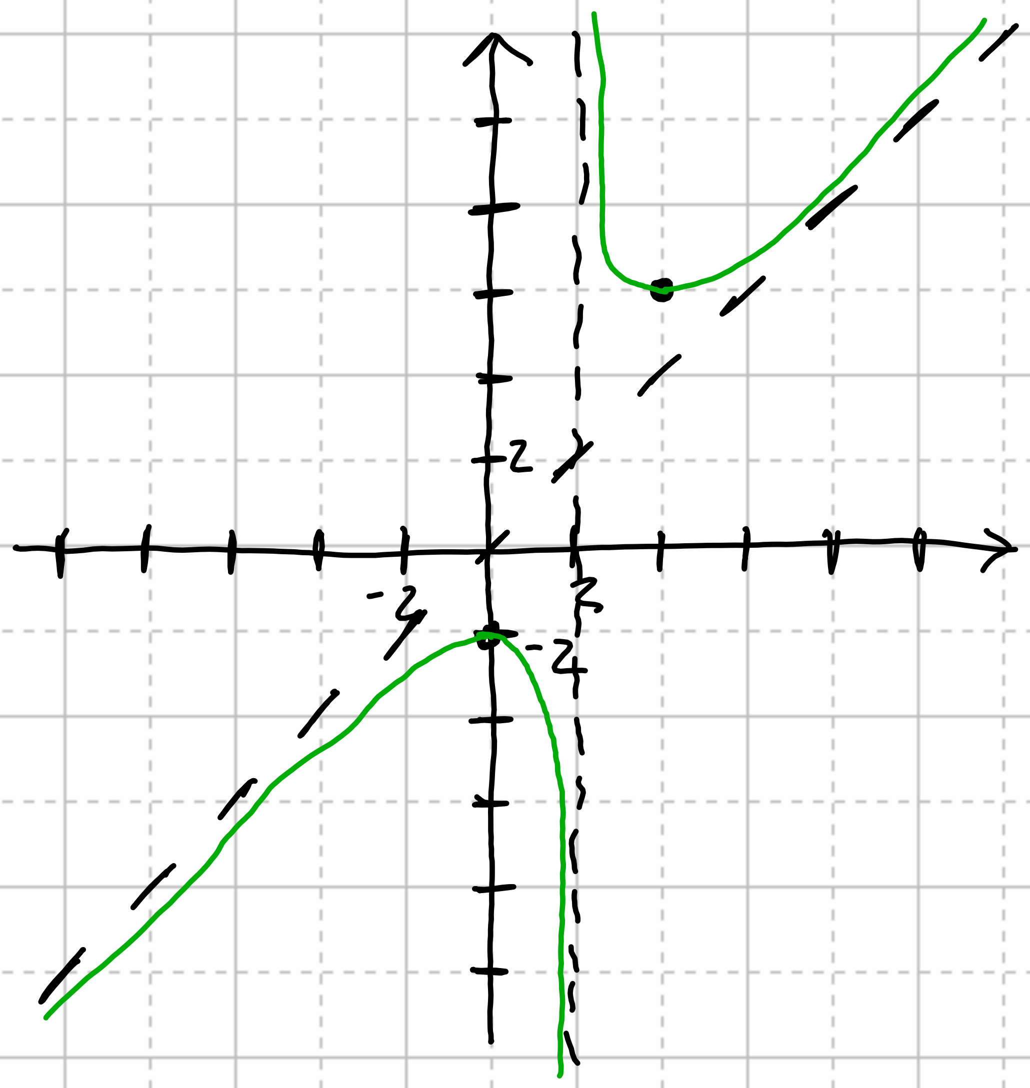
Graphical Analysis Techniques

- Items we can find from $f(x)$
  - → Domain / Range
  - → x- + y-intercepts
  - → Asymptotes (Vertical / Horizontal / Slant) (End Behavior)
- Items we can find from $f'(x)$
  - → Critical #'s
  - → Increasing / Decreasing Intervals
  - → Relative Extrema (Maxima / Minima)
- Items we can find from $f''(x)$
  - → Concavity (Points of Inflection / Up / Down)
  - → Relative Extrema

EX 1 → $f(x) = \frac{x^2-2x+4}{x-2}$



D: all x, $x \neq 2$

x-int: NONE

y-int: $(0,-2)$

V.A.: $x=2$

H.A.: NONE

S.A.: $\begin{array}{r} x + \frac{4}{x-2} \\ x-2 \overline{) x^2-2x+4} \\ \underline{-(x^2-2x)} \\ 0+4 \end{array} \Rightarrow y=x$

$f'(x) = \frac{(x-2)(2x-2)-(x^2-2x+4)(1)}{(x-2)^2}$

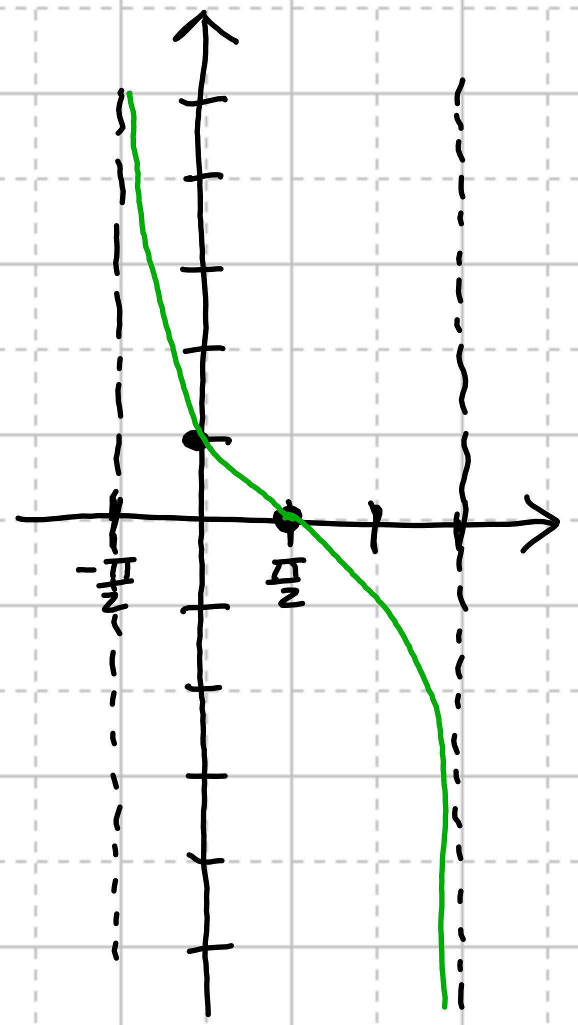
$= \frac{2x^2-4x-2x+4-x^2+2x-4}{(x-2)^2} = \frac{x^2-4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2} = 0$

+		−		−		+
$f'(-1)$	0	$f'(1)$	2	$f'(3)$	4	$f'(5)$
	↑ max				↑ min → $f(4)=6$	

$f(x)$ inc $(-\infty,0),(4,\infty)$

dec $(0,2),(2,4)$

EX2 → $f(x) = \dfrac{\cos x}{1+\sin x}, \quad -\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$



V.A.: $1+\sin x = 0$

$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}, -\frac{\pi}{2}$

X-int: $\cos x = 0 \Rightarrow x = \frac{\pi}{2} \Rightarrow \left(\frac{\pi}{2}, 0\right)$

y-int: $\dfrac{\cos(0)}{1+\sin(0)} = \dfrac{1}{1+0} = 1 \Rightarrow (0,1)$

$$f'(x) = \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1(\sin^2 x + \cos^2 x)}{(1+\sin x)^2} = \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1\cancel{(\sin x + 1)}}{(1+\sin x)^{\cancel{2}}} = \frac{-1}{1+\sin x}$$

$-$

$-\frac{\pi}{2}$ $f'(0)$ $\frac{3\pi}{2}$

$f(x)$ dec on $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$$f''(x) = \frac{(1+\sin x)(0) - (-1)(\cos x)}{(1+\sin x)^2} = \frac{\cos x}{(1+\sin x)^2} = 0 \Rightarrow x = \frac{\pi}{2}$$

$+$ $-$

$-\frac{\pi}{2}$ $f''(0)$ $\frac{\pi}{2}$ $f''(\pi)$ $\frac{3\pi}{2}$

$f(x)$ concave up on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

concave down on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

HW: p. 215 → 1-6, 12-30 mult. 6