Related Rates

- Measuring the rate of change of two related variables w/ respect to time
  - ↳ implicit differentiation is used in same manner

EX1 → $4x^2 - 3y^2 = 1$, $\frac{dx}{dt} = -3$, $(1,1)$, $\frac{dy}{dt} = ?$

$$8x \cdot \frac{dx}{dt} - 6y \cdot \frac{dy}{dt} = 0$$

$$8(1) \cdot (-3) - 6(1) \cdot \frac{dy}{dt} = 0$$

$$-24 - 6\frac{dy}{dt} = 0$$

$$-6\frac{dy}{dt} = 24$$

$$\frac{dy}{dt} = -4$$

EX2 → $5xy = 10$, $(1,2)$, $\frac{dx}{dt} = 7$, $\frac{dy}{dt} = ?$

$$5x \cdot \frac{dy}{dt} + y \cdot 5 \cdot \frac{dx}{dt} = 0$$

$$5x\frac{dy}{dt} + 5y\frac{dx}{dt} = 0$$

$$5(1)\frac{dy}{dt} + 5(2)(7) = 0$$

$$5\frac{dy}{dt} = -70$$

$$\frac{dy}{dt} = -14$$

EX3 → What rate is the area of a circle changing when the radius is at 5 ft and is increasing at a rate of 2 ft/s?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (5\text{ ft})(2\text{ ft/s})$$

$$\frac{dA}{dt} = 20\pi \text{ ft}^2/\text{s}$$

EX4 → What rate is the area of a rectangle changing when the length is 4 ft, the width is 3 ft and both are changing at a rate of 7 ft/min?

$$A = l \cdot w$$

$$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + w \cdot \frac{dl}{dt}$$

$$\frac{dA}{dt} = (4\text{ ft})(7\text{ ft/min}) + (3\text{ ft})(7\text{ ft/min})$$

$$\frac{dA}{dt} = 49 \text{ ft}^2/\text{min}$$

<u>HW</u>: p. 154 → 1-8, 15, 18