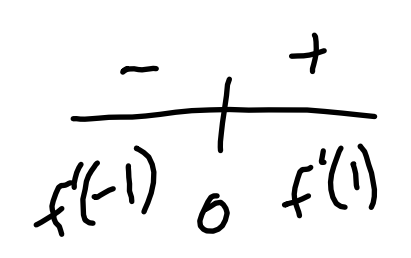


Inverse Functions

EX $\rightarrow y = 3x - 7 \Rightarrow f^{-1}(x) \rightarrow x = 3y - 7$
 $\frac{x+7}{3} = y$

- Checking to see if a function is 1-to-1
 - \rightarrow passes horizontal line test
 - \rightarrow function should be strictly monotonic (constantly increasing/decreasing)

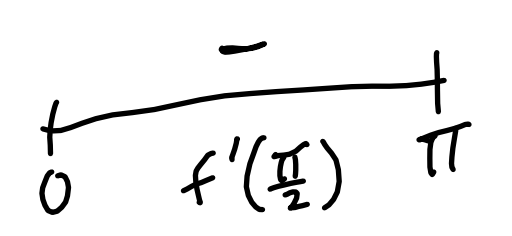
EX1 $\rightarrow y = 6x^2 + 7$
 $y' = 12x = 0$
 $x = 0$



\Rightarrow NOT 1-to-1
(would be monotonic on $(-\infty, 0)$ OR $(0, \infty)$)

EX2 \rightarrow Is $f(x) = \cos x$ 1-to-1 on $(0, \pi)$?

$f'(x) = -\sin x = 0$
 $x = 0, \pi$



$\Rightarrow f(x)$ is 1-to-1

HW : p. 347 → 9-12, 23-28, 37-51 odd

②④ $f(x) = \cos\left(\frac{3}{2}x\right)$
 $f'(x) = -\frac{3}{2} \sin\left(\frac{3}{2}x\right) = 0$
 $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \dots$

②⑦ $f(x) = 2 - x - x^3$
 $f'(x) = -1 - 3x^2 = 0$
 ALWAYS NEGATIVE
 \downarrow
YES

②⑧ $f(x) = (x+a)^3 + b$
 $f'(x) = 3(x+a)^2$
 \downarrow
 ALWAYS POSITIVE
 \downarrow
YES

$\frac{-}{-} \frac{+}{+} \frac{-}{-} \frac{+}{+} \Rightarrow$ NO

④① $f(x) = \frac{x}{\sqrt{x^2+7}}$
 $x = \frac{y}{\sqrt{y^2+7}}$
 $x^2 = \frac{y^2}{y^2+7}$

$x^2(y^2+7) = y^2$
 $x^2y^2 + 7x^2 = y^2$
 $7x^2 = y^2 - x^2y^2$
 $7x^2 = y^2(1-x^2)$
 $y^2 = \frac{7x^2}{1-x^2}$

$y = \sqrt{\frac{7x^2}{1-x^2}}$

④⑤ B) $y = -0.35x + 80$
 $\dot{x} = -0.35y + 80$
 $\frac{x-80}{-0.35} = y$

C) $x < 80$

D) $73 = -0.35x + 80$
 $-7 = -0.35x$
 $x = 20$

- Derivative of Inverse Functions

→ Let $f(x)$ be a differentiable function & $g(x)$ be its inverse

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0$$

EX1 → $f(x) = 3x + 4 \rightarrow f'(x) = 3$

$$f(3) = 13 \Rightarrow g(13) = 3$$

$$g'(13) = \frac{1}{f'(g(13))} = \frac{1}{f'(3)} = \boxed{\frac{1}{3}}$$

EX2 → $f(x) = 6x^2 + 7 \rightarrow f'(x) = 12x$

$$f(4) = 103 \Rightarrow g(103) = 4$$

$$g'(103) = \frac{1}{f'(4)} = \boxed{\frac{1}{48}}$$

HW: p. 348 → 59-65 odd,
71-89 odd