Inverse Functions

EX → $y = 3x - 7 \Rightarrow f^{-1}(x) \rightarrow x = 3y - 7$

$$\frac{x+7}{3} = y$$

- Checking to see if a function is 1-to-1
  - → passes horizontal line test
  - → function should be strictly monotonic (constantly increasing/decreasing)

EX1 → $y = 6x^2 + 7$

$y' = 12x = 0$

$x = 0$

$$\begin{array}{ccc} - & & + \\ \hline f'(-1) & 0 & f'(1) \end{array}$$

$\Rightarrow$ NOT 1-to-1

(would be monotonic on $(-\infty, 0)$ OR $(0, \infty)$)

EX2 → Is $f(x) = \cos x$ 1-to-1 on $(0, \pi)$?

$f'(x) = -\sin x = 0$

$x = 0, \pi$

$$\begin{array}{ccc} & - & \\ \hline 0 & f'(\frac{\pi}{2}) & \pi \end{array}$$

$\Rightarrow f(x)$ is 1-to-1

HW: p. 347 → 9-12, 23-28, 37-51 odd

(24) $f(x) = \cos\left(\frac{3}{2}x\right)$

$f'(x) = -\frac{3}{2}\sin\left(\frac{3}{2}x\right) = 0$

$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$

$-\quad +\quad -\quad +$

$-\frac{2\pi}{3} \quad f'\left(\frac{\pi}{3}\right) \quad 0 \quad f'\left(\frac{\pi}{3}\right) \quad \frac{2\pi}{3}$

$\Rightarrow$ NO

(27) $f(x) = 2 - x - x^3$

$f'(x) = -1 - 3x^2 = 0$

ALWAYS NEGATIVE

$\downarrow$

YES

(28) $f(x) = (x+a)^3 + b$

$f'(x) = 3(x+a)^2$

$\downarrow$

ALWAYS POSITIVE

$\downarrow$

YES

(41) $f(x) = \frac{x}{\sqrt{x^2+7}}$

$x = \frac{y}{\sqrt{y^2+7}}$

$x^2 = \frac{y^2}{y^2+7}$

$x^2(y^2+7) = y^2$

$x^2y^2 + 7x^2 = y^2$

$7x^2 = y^2 - x^2y^2$

$7x^2 = y^2(1-x^2)$

$y^2 = \frac{7x^2}{1-x^2}$

$y = \sqrt{\frac{7x^2}{1-x^2}}$

(45) B) $y = -0.35x + 80$

$x = -0.35y + 80$

$\frac{x - 80}{-0.35} = y$

C) $x < 80$

D) $73 = -0.35x + 80$

$-7 = -0.35x$

$x = 20$

- Derivative of Inverse Functions

→ Let $f(x)$ be a differentiable function & $g(x)$ be its inverse

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) = 0$$

<u>EX1</u> → $f(x) = 3x+4 \rightarrow f'(x) = 3$

$f(3) = 13 \Rightarrow g(13) = 3$

$g'(13) = \frac{1}{f'(g(13))} = \frac{1}{f'(3)} = \boxed{\frac{1}{3}}$

<u>EX2</u> → $f(x) = 6x^2+7 \rightarrow f'(x) = 12x$

$f(4) = 103 \Rightarrow g(103) = 4$

$g'(103) = \frac{1}{f'(4)} = \boxed{\frac{1}{48}}$

<u>HW</u>: p. 348 → 59–65 odd, 71–89 odd