Inverse Trig Functions

→ Derivatives of Inverse Trig Functions

→ $\frac{d}{dx}\left[\sin^{-1} u\right] = \frac{u'}{\sqrt{1-u^2}}$

→ $\frac{d}{dx}\left[\cos^{-1} u\right] = \frac{-u'}{\sqrt{1-u^2}}$

→ $\frac{d}{dx}\left[\tan^{-1} u\right] = \frac{u'}{1+u^2}$

→ $\frac{d}{dx}\left[\sec^{-1} u\right] = \frac{u'}{|u|\sqrt{u^2-1}}$

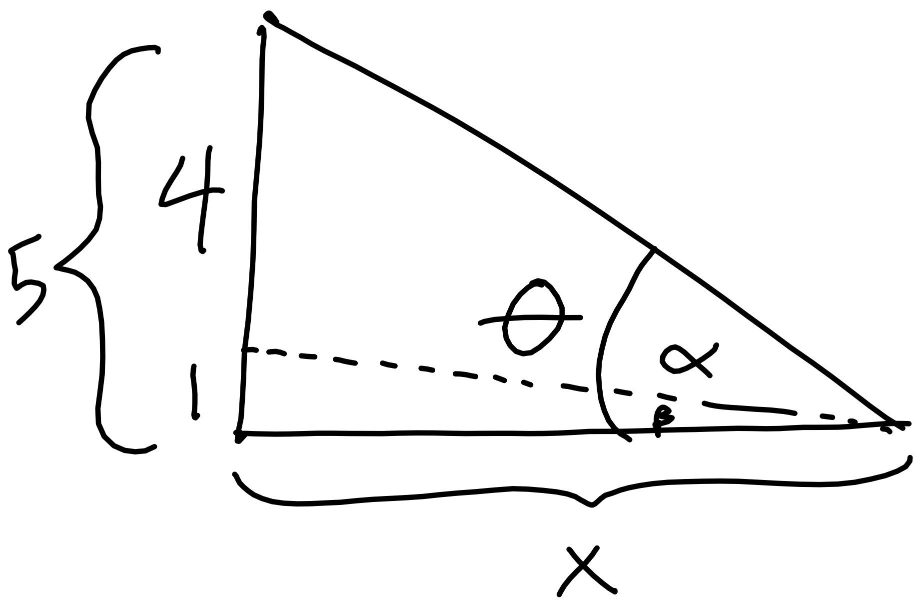
→ $\frac{d}{dx}\left[\csc^{-1} u\right] = \frac{-u'}{|u|\sqrt{u^2-1}}$

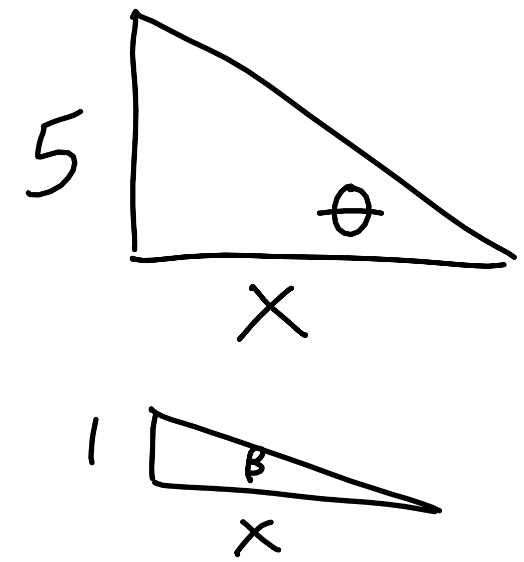
→ $\frac{d}{dx}\left[\cot^{-1} u\right] = \frac{-u'}{1+u^2}$

<u>EX1</u> → $\frac{d}{dx}\left[\cos^{-1} 4x\right] = \frac{-4}{\sqrt{1-(4x)^2}} = \frac{-4}{\sqrt{1-16x^2}}$

<u>EX2</u> → $\frac{d}{dx}\left[\tan^{-1}(12x^2)\right] = \frac{24x}{1+(12x^2)^2} = \frac{24x}{1+144x^4}$

EX3 → (p. 375, EX. 7)





$\tan\theta = \frac{5}{x}$

$\theta = \tan^{-1}\left(\frac{5}{x}\right)$

$\tan\beta = \frac{1}{x}$

$\beta = \tan^{-1}\left(\frac{1}{x}\right)$

$$\alpha = \theta - \beta = \tan^{-1}\left(\frac{5}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$$

$$\alpha' = \frac{\frac{-5}{x^2}}{1+\left(\frac{5}{x}\right)^2} - \frac{\frac{-1}{x^2}}{1+\left(\frac{1}{x}\right)^2} = \frac{\frac{-5}{x^2}}{1+\frac{25}{x^2}} + \frac{\frac{1}{x^2}}{1+\frac{1}{x^2}} = \frac{-5}{x^2\left(1+\frac{25}{x^2}\right)} + \frac{1}{x^2\left(1+\frac{1}{x^2}\right)} = \frac{-5}{x^2+25} + \frac{1}{x^2+1}$$

$$= \frac{-5(x^2+1) + 1(x^2+25)}{(x^2+25)(x^2+1)} = \frac{-4x^2+20}{(x^2+25)(x^2+1)} = 0$$

$-4x^2+20=0$

$4x^2=20$

$x=\sqrt{5}$

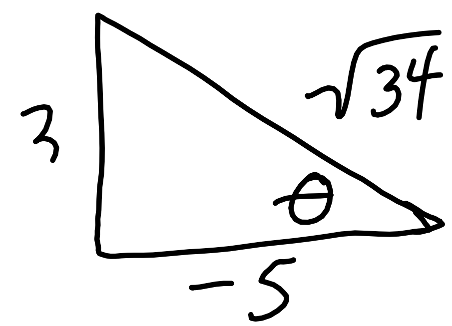
$\alpha = \tan^{-1}\left(\frac{5}{\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\alpha = 0.729 \approx 41.810^\circ$

- Evaluating Expressions

→ DRAW RIGHT TRIANGLES!

EX1 → $\sec\left(\arctan\left(-\frac{3}{5}\right)\right) = \frac{\sqrt{34}}{-5}$



<u>HW</u>: p. 377 → 5-11 odd, 17-27 odd, 31, 33, 42-63 mult. 7

(33) $\sin^{-1}(\sqrt{2x}) = \cos^{-1}(\sqrt{x})$

$$\sin(\sin^{-1}(\sqrt{2x})) = \sin(\cos^{-1}(\sqrt{x}))$$

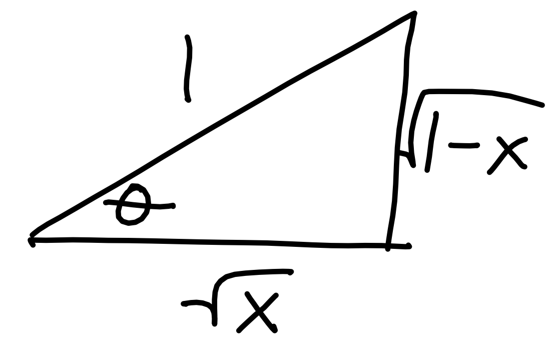
$$\sqrt{2x} = \sin(\cos^{-1}(\sqrt{x}))$$

$$\sqrt{2x} = \sqrt{1-x}$$

$$2x = 1-x$$

$$3x = 1$$

$$x = \frac{1}{3}$$



$1^2 = (\sqrt{x})^2 + ?^2$

$1 = x + ?^2$

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27

33

49

56

- Integrating Inverse Trig Functions

(u is a differentiable function, a is greater than 0)

$\rightarrow \int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + C$

$\rightarrow \int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

$\rightarrow \int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$

EX1 $\rightarrow \int \frac{1}{\sqrt{9-x^2}}\,dx = \sin^{-1}\left(\frac{x}{3}\right) + C$

EX2 $\rightarrow \displaystyle\int \frac{1}{\sqrt{e^{4x}-1}}\,dx$

$u = e^{2x}$

$du = 2e^{2x}\,dx \Rightarrow \dfrac{du}{2e^{2x}} = dx \Rightarrow \dfrac{du}{2u} = dx$

$$= \int \frac{1}{\sqrt{u^2-1}} \cdot \frac{1}{2u} \cdot du = \int \frac{1}{\sqrt{u^2-1}} \cdot \frac{1}{2} \cdot \frac{1}{u} \cdot du = \frac{1}{2}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= \frac{1}{2}\cdot\frac{1}{1}\sec^{-1}\frac{|u|}{1} + C = \boxed{\frac{1}{2}\sec^{-1}\left(e^{2x}\right) + C}$$

EX3 $\rightarrow \displaystyle\int \frac{1}{x^2-4x+7}\,dx$

$$= \int \frac{1}{(x^2-4x+4)+3}\,dx$$

$$= \int \frac{1}{(x-2)^2+3}\,dx = \boxed{\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-2}{\sqrt{3}}\right) + C}$$

HW: p. 385 → 3-39 mult. 3

(30) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

$u = \sin x$
$du = \cos x\,dx$

$$= \int_{x=0}^{x=\pi/2} \frac{1}{1+u^2} \cdot du$$

$$= \frac{1}{1}\tan^{-1}\frac{u}{1}\Big|_{x=0}^{x=\frac{\pi}{2}} = \tan^{-1}(\sin x)\Big|_0^{\pi/2} = \tan^{-1}\left(\sin\left(\tfrac{\pi}{2}\right)\right) - \tan^{-1}(\sin(0)) = \tan^{-1}(1) - \cancel{\tan^{-1}(0)} \to 0$$

$$= \frac{\pi}{4}$$

(33) $\displaystyle\int \frac{2x}{x^2+6x+13}\,dx = \int \frac{2x}{(x^2+6x+9)+4}\,dx = \int \frac{2x+6-6}{(x^2+6x+9)+4}\,dx$

$$= \int \frac{2x+6}{(x^2+6x+9)+4}\,dx - \int \frac{6}{(x^2+6x+9)+4}\,dx = \int \frac{1}{u_1}\cdot du_1 - \int \frac{6}{(x+3)^2+4}\,dx = \ln|u_1| - 6\int \frac{1}{u_2^2+4}\,du_2$$

$u_1 = x^2+6x+13$
$du_1 = 2x+6\,dx$

$u_2 = x+3$
$du_2 = dx$

$$= \ln|x^2+6x+13| - 6\cdot\frac{1}{2}\cdot\tan^{-1}\frac{u_2}{2}$$

$$= \ln|x^2+6x+13| - 3\tan^{-1}\left(\frac{x+3}{2}\right) + C$$

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$$\int \frac{2}{\sqrt{-x^2+4x}}\,dx$$

$$= \int \frac{2}{\sqrt{-(x^2-4x)}}\,dx$$

$$= \int \frac{2}{\sqrt{-(x^2-4x+4-4)}}\,dx$$

$$= \int \frac{2}{\sqrt{4-(x^2-4x+4)}}\,dx$$

$$= \int \frac{2}{\sqrt{4-(x-2)^2}}\,dx$$

$$= 2\cdot\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

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$$\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx$$

$$= \int_2^3 \frac{2x-4+1}{\sqrt{4x-x^2}}\,dx$$

$$= \int_2^3 \frac{2x-4}{\sqrt{4x-x^2}}\,dx + \int_2^3 \frac{1}{\sqrt{4x-x^2}}\,dx$$

$u = 4x-x^2$

$du = (4-2x)\,dx$

$-du = (2x-4)\,dx$

$$\int_{x=2}^{x=3} \frac{-1}{\sqrt{u}}\,du + \int_2^3 \frac{1}{\sqrt{4-(x-2)^2}}$$

$$= -2u^{1/2}\Big|_{x=2}^{x=3} + \sin^{-1}\left(\frac{x-2}{2}\right)\Big|_2^3$$

$$= -2\sqrt{4x-x^2} + \sin^{-1}\left(\frac{x-2}{2}\right)\Big|_2^3$$

$$= \left[-2\sqrt{3} + \sin^{-1}\left(\tfrac{1}{2}\right)\right] - \left[-2\sqrt{4} + \overset{0}{\sin^{-1}(0)}\right]$$

$$= -2\sqrt{3} + \frac{\pi}{6} + 4$$

HW: p. 378 → 41-65 odd, 89-91

p. 385 → 1-41 odd, 63-67 odd