L'Hôpital's Rule

→ Let f & g be functions that are differentiable on open interval (a,b) containing c, except possibly at c itself. Assume $g'(x) \neq 0$ for all x in (a,b), except possibly at c itself. If the limit of $\frac{f(x)}{g(x)}$ as x approaches c produces the indeterminate form $\frac{0}{0}$, $\frac{\infty}{\infty}$, or $\frac{-\infty}{\infty}$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists or is infinite.

EX1 → $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x} = \dfrac{e^{0}-1}{0} = \dfrac{1-1}{0} = \dfrac{0}{0} \Rightarrow \lim\limits_{x\to 0} \dfrac{2e^{2x}}{1} = 2\,\overset{1}{\cancel{e^{0}}} = \boxed{2}$

EX2 → $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = \dfrac{0}{0} \Rightarrow \lim\limits_{x\to 0} \dfrac{\cos x}{1} = \cos(0) = \boxed{1}$

EX3 → $\lim\limits_{x\to \infty} \dfrac{\ln x}{x} = \dfrac{\infty}{\infty} \Rightarrow \lim\limits_{x\to \infty} \dfrac{\frac{1}{x}}{1} = \lim\limits_{x\to \infty} \dfrac{1}{x} = \boxed{0}$

- Rule can be applied as many times as necessary,

EX → $\lim\limits_{x\to -\infty} \dfrac{x^{2}}{e^{-x}} = \dfrac{\infty}{\infty} \Rightarrow \lim\limits_{x\to -\infty} \dfrac{2x}{-e^{-x}} = \dfrac{-\infty}{-\infty} \Rightarrow \lim\limits_{x\to -\infty} \dfrac{2}{e^{-x}} = \dfrac{2}{\infty} = \boxed{0}$

HW:

p. 574 → 5-35 mult. 5