Motion of a Particle/Rates of Change

- Instantaneous rate of change

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

This is the definition of a derivative; therefore to find an instantaneous rate of change, we must take the derivative of the function

- Motion of Particle Functions
  - → Position function ⇒ $s(t)$ (w/ respect to time t) (measured in units of length) (can be positive or negative) (displacement $= s_f - s_i$)
  - → Velocity function ⇒ $v(t) = \frac{\text{length}}{\text{time}} = s'(t)$
  - * Velocity is a vector function, meaning that it is a measurement w/ a direction (pos. or neg.) Speed is only a measurement. Therefore, speed $= |v(t)|$
  - → Acceleration function ⇒ $a(t) = \frac{v(t)}{\text{time}} = v'(t) = s''(t)$
  - → Jerk function ⇒ $j(t) = \frac{a(t)}{\text{time}} = a'(t) = v''(t) = s'''(t)$

- Free-Fall Constants

$\rightarrow g = 32\ ft/s^2 = 9.8\ m/s^2$ $\left(s = \frac{1}{2}gt^2\right)$

(W/ vertical motion, maximum height is reached when $v=0$)

EX → A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in sec.

A) Find the displacement of the particle during the first 2 seconds

$s(2) - s(0) = -1 - 3 = \boxed{-4\ m}$

B) Find the average velocity of the particle during the first 4 seconds.

$V_{avg} = \frac{s(4) - s(0)}{4} = \frac{3-3}{4} = \boxed{0\ m/s}$

C) Find the instantaneous velocity of the particle when $t=4$

$v(t) = s'(t) = 2t - 4$

$v(4) = 2(4) - 4 = \boxed{4\ m/s}$

D) Find the acceleration when $t=4$

$a(t) = v'(t) = 2 \Rightarrow \boxed{a(4) = 2 \ m/s^2}$

E) Describe the motion of the particle. At what values of t does the particle change directions?

$v = 0$ ~~$v(0)$~~

$v(t) = 2t - 4 = 0$ @ $t = 2$

$[0, 2)$ moving left

$(2, \infty)$ right

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$s(t) = -16t^2 - 22t + 220$

$v(t) = -32t - 22$

$v(3) = -32(3) - 22$

$= -96 - 22 = -118 \text{ ft/s}$

$v(2) = -32(2) - 22$

$= -64 - 22 = -86 \text{ ft/s}$

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$s(t) = -4.9t^2 + 120t + 0$

$v(t) = -9.8t + 120$

$v(5) = -9.8(5) + 120 = 71 \text{ m/s}$

$v(10) = -9.8(10) + 120 = 22 \text{ m/s}$

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$s(t) = -4.9t^2 + \cancel{0t}^{\,0} + s_0 = 0$

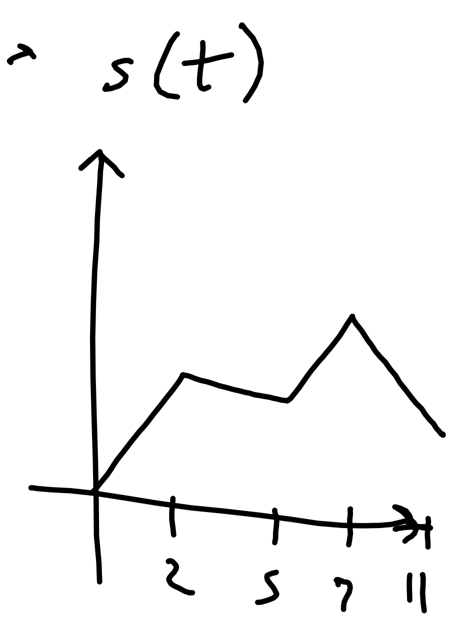
$s(6.8) = 0 = -4.9(6.8)^2 + s_0$

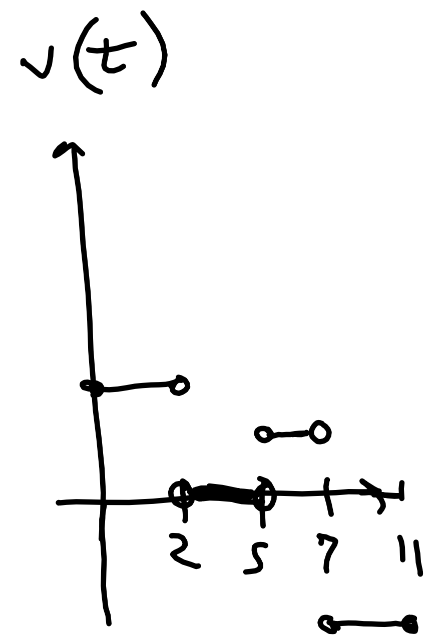
$s_0 = 4.9(6.8)^2 = 226.576 \text{ m}$

<u>HW</u>: p. 117 → 93 – 96

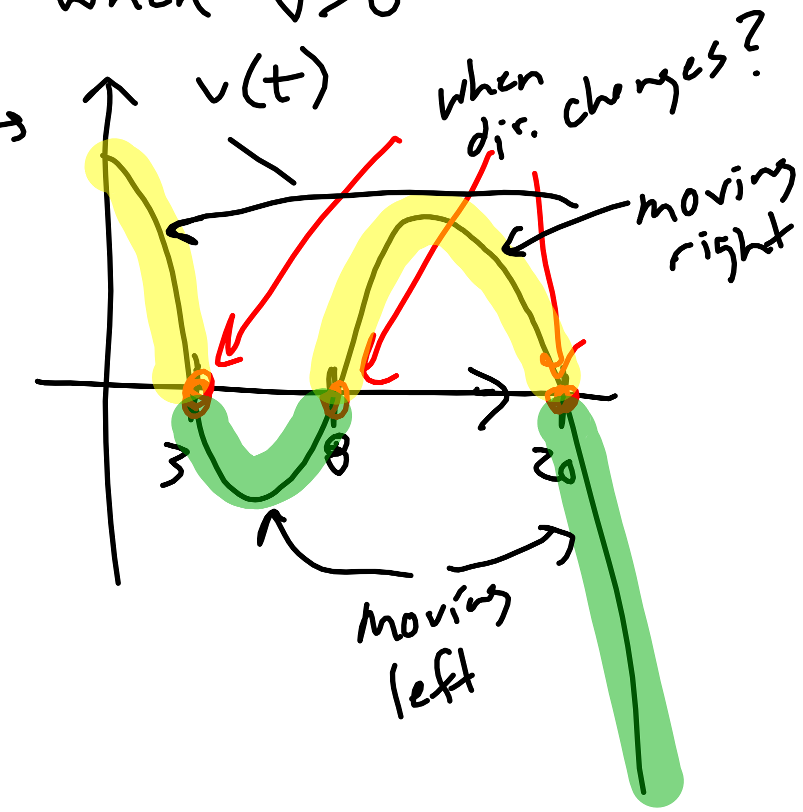
- Analyzing Position / Velocity Graphs
  - Position Graphs
    - Find slopes over certain intervals (indicates constant velocity)
    - If position function is given & is quadratic, cubic, etc., then velocity function must be found in order to graph (take derivative)
  - Velocity graphs
    - Remember that a particle changes direction @ $v=0$, is moving to the left when $v<0$, & is moving to the right when $v>0$

EX →





EX →



- When velocity & acceleration have same sign → object speeds up
- When velocity & acceleration have diff. signs → object slows down

(24) $v(t) = 2t^3 - 9t^2 + 12t - 5$

$a(t) = 6t^2 - 18t + 12 = 6(t^2 - 3t + 2) = 0$

$6(t-2)(t-1) = 0$

$t = 1, 2$

$v(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5$

$= 2 - 9 + 12 - 5 = 0 \text{ m/s}$

$v(2) = 2(2)^3 - 9(2)^2 + 12(2) - 5$

$= 16 - 36 + 24 - 5 = -1 \text{ m/s}$

$$\boxed{\begin{array}{l} \text{Speed}(2) = 1 \text{ m/s} \\ \text{Speed}(1) = 0 \text{ m/s} \end{array}}$$

(20) $s(t) = -t^3 + 7t^2 - 14t + 8$

$v(t) = -3t^2 + 14t - 14 = 0$

AT REST

$t = \frac{-\sqrt{7} \pm 7}{3} = \underline{1.451} + \underline{3.215}$

$[0, 1.451) \rightarrow$ left

$(1.451, 3.215) \rightarrow$ right

$(3.215, \infty) \rightarrow$ left

(15) $s(t) = 24t - 4.9t^2$

$v(t) = 24 - 9.8t = 0$

$\frac{\cancel{9.8}t}{\cancel{9.8}} = \frac{24}{9.8}$

$t = 2.449$

$s(2.449) = 24(2.449) - 4.9(2.449)^2$

$= 29.388$ m