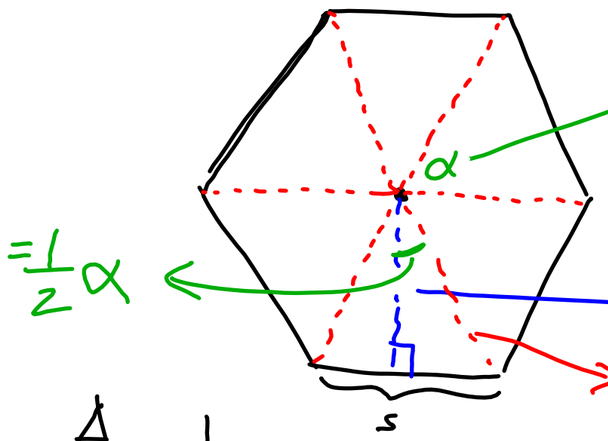
Area of Regular Polygons

- regular → all sides & angles are equal



central angle (angle between 2 radii, = to $\frac{360}{n}$)
n = # of sides

$= \frac{1}{2}\alpha$

apothem (center to side)

radius (center to vertex)

$A = \frac{1}{2}a \cdot s + \frac{1}{2}a \cdot s + \frac{1}{2}a \cdot s + \frac{1}{2}a \cdot s + \frac{1}{2}a \cdot s + \frac{1}{2}a \cdot s$

$A = \frac{1}{2}a(\underbrace{s+s+s+s+s+s}_{\text{Perimeter}})$

$\boxed{A = \frac{1}{2}aP}$

Names of Polygons

4 → quadrilateral
5 → pentagon
6 → hexagon
7 → heptagon / septagon
8 → octagon
9 → nonagon
10 → decagon
12 → dodecagon
n → n-gon

EX → heptagon, $a=10$, $s=5$

$$A=\frac{1}{2}(10)(7.5)$$
$$=\frac{1}{2}(10)(35)$$
$$=175\ u^2$$

EX → dodecagon, $a=8$, $s=3$

$$A=\frac{1}{2}(8)(12.3)$$
$$=\frac{1}{2}(8)(36)$$
$$=144\ u^2$$

EX → nonagon, $a=7$, $s=4$

$$A=\frac{1}{2}(7)(9.4)$$
$$=\frac{1}{2}(7)(36)$$
$$=126\ u^2$$

EX → dodecagon, $a = 8$, $s = 10$

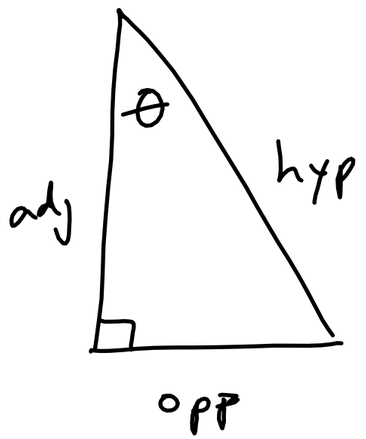
$$A = \frac{1}{2}(8)(12 \cdot 10)$$
$$= \frac{1}{2}(8)(120)$$
$$= 480 \text{ u}^2$$

EX → nonagon, $a = 5$, $s = 4$

$$A = \frac{1}{2}(5)(9 \cdot 4)$$
$$= \frac{1}{2}(5)(36)$$
$$= 90 \text{ u}^2$$

EX → heptagon, $a = 6$, $s = 12$

$$A = \frac{1}{2}(6)(7 \cdot 12)$$
$$= \frac{1}{2}(6)(84)$$
$$= 252 \text{ u}^2$$



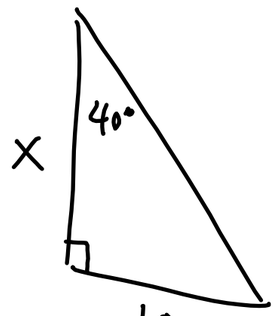
$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

(*) Using trig will be necessary to find missing pieces in order to eventually solve for the area

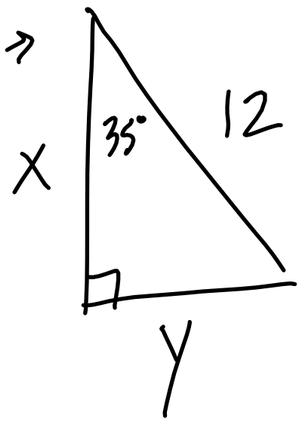
EX →



$\tan 40 = \frac{10}{X}$

$X = \frac{10}{\tan 40} = 11.9$

EX →



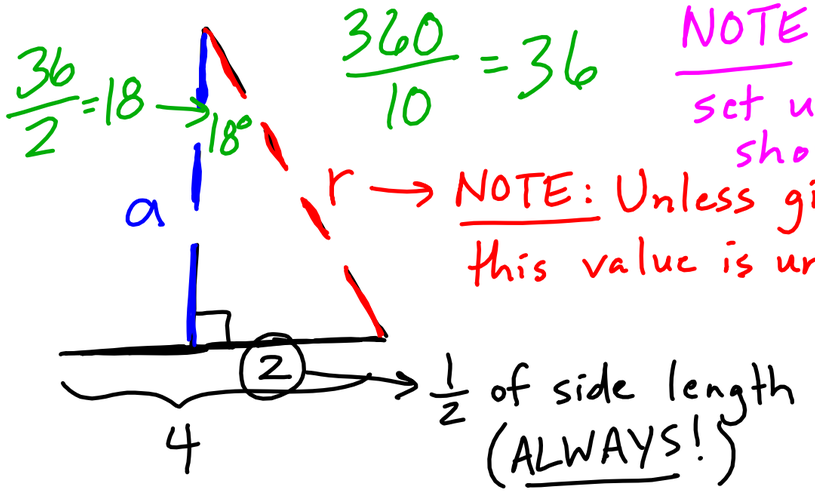
$\cos 35 = \frac{X}{12}$

$X = 9.830$

$\sin 35 = \frac{Y}{12}$

$Y = 6.883$

EX → (7) p. 646 decagon, $s = 4$ yd.



$\frac{360}{10} = 36$

NOTE: ALWAYS set up as shown!

NOTE: Unless given, this value is unnecessary

$\frac{1}{2}$ of side length (ALWAYS!)

$\tan 18° = \frac{2}{a}$

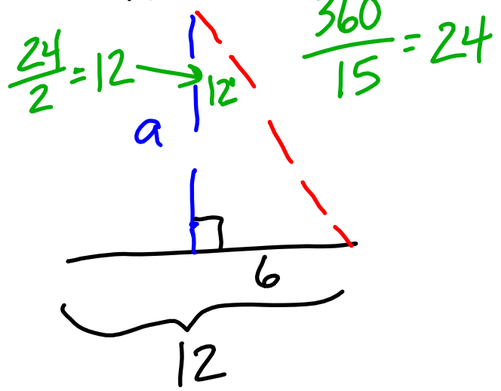
$a = \frac{2}{\tan 18°} = 6.155$

$A = \frac{1}{2}(6.155)(10 \cdot 4)$

$A = 123.1 \text{ yd}^2$

EX → (13) p. 646 15-gon, $P = 180$ cm

$\frac{180}{15} = 12$ cm



$\frac{360}{15} = 24$

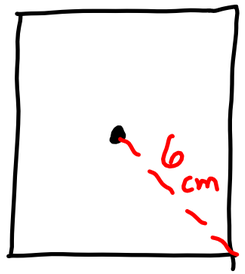
$\tan 12° = \frac{6}{a}$

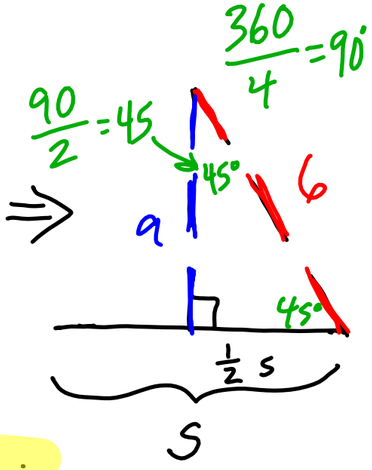
$a = \frac{6}{\tan 12°} = 28.228$

$A = \frac{1}{2}(28.228)(180)$

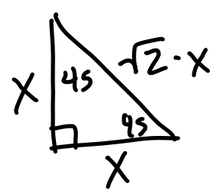
$A = 2540.52 \text{ cm}^2$

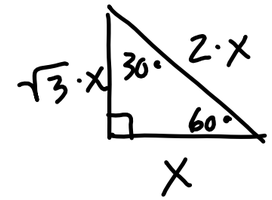
EX → (21) p. 632





REMEMBER:





$6 = \sqrt{2} \cdot a$

$a = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

OR $A = (6\sqrt{2})^2 = 72 \text{ cm}^2$

$\frac{1}{2}s = 3\sqrt{2} \Rightarrow s = 6\sqrt{2}$

$A = \frac{1}{2}(3\sqrt{2})(4 \cdot 6\sqrt{2})$

$A = 72 \text{ cm}^2$

HW: p. 632 → 11-25, 44, 48, 49

p. 646 → 6-12 even