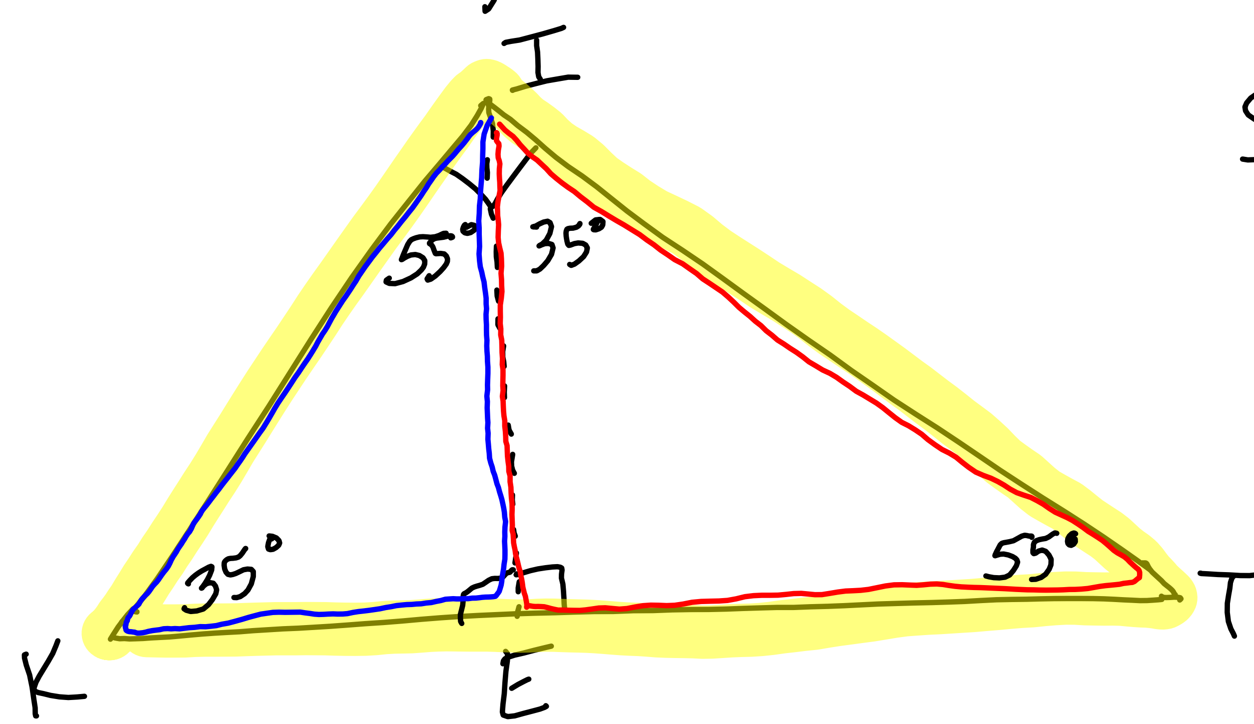
# Similarity in Right Triangles

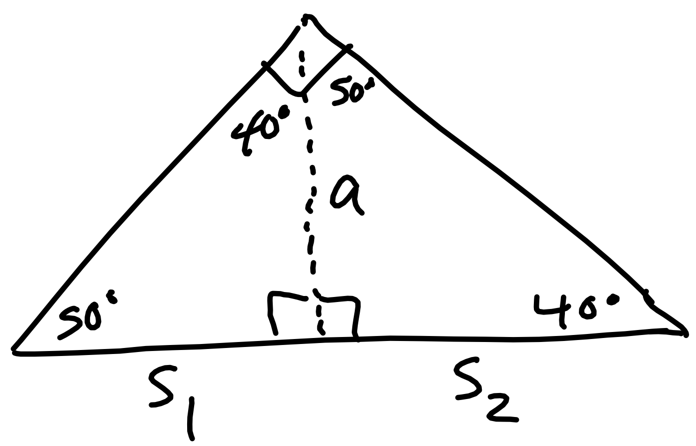
- Altitudes in Right △'s Theorem → Altitude creates 3 similar △'s




Similarity Statement

△KIT ~ △KEI ~ △IET

- Corollary 1 (Altitudes Rule) → altitude is proportional to segments of hypotenuse




$$\frac{\text{short leg}}{\text{long leg}} = \frac{s_1}{a} = \frac{a}{s_2} \Rightarrow a^2 = s_1 \cdot s_2$$

$$a = \sqrt{s_1 \cdot s_2}$$ → Geometric Mean

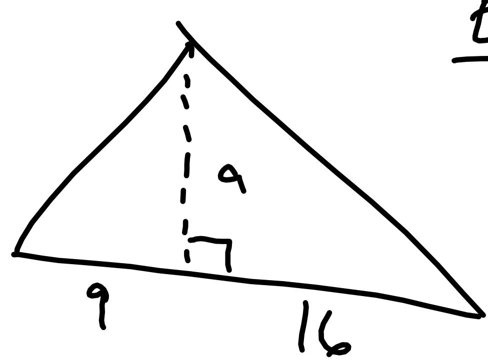
(*) Geometric mean → square root of the product of 2 #'s

EX → 4, 25 ⇒ $\sqrt{4 \cdot 25} = \sqrt{100} = 10$

EX → 3, 9 ⇒ $\sqrt{3 \cdot 9} = \sqrt{27} = 3\sqrt{3}$

EX → 7, 12 ⇒ $\sqrt{7 \cdot 12} = \sqrt{84} = \sqrt{4} \cdot \sqrt{21} = 2\sqrt{21}$

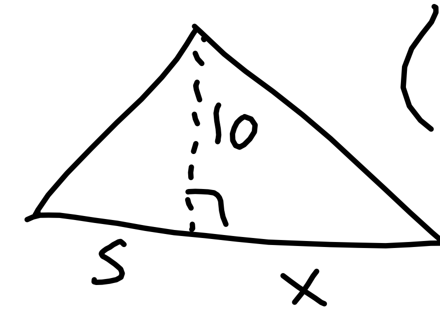
EX →




$a = \sqrt{9 \cdot 16}$

$a = 3 \cdot 4 = 12$

EX →

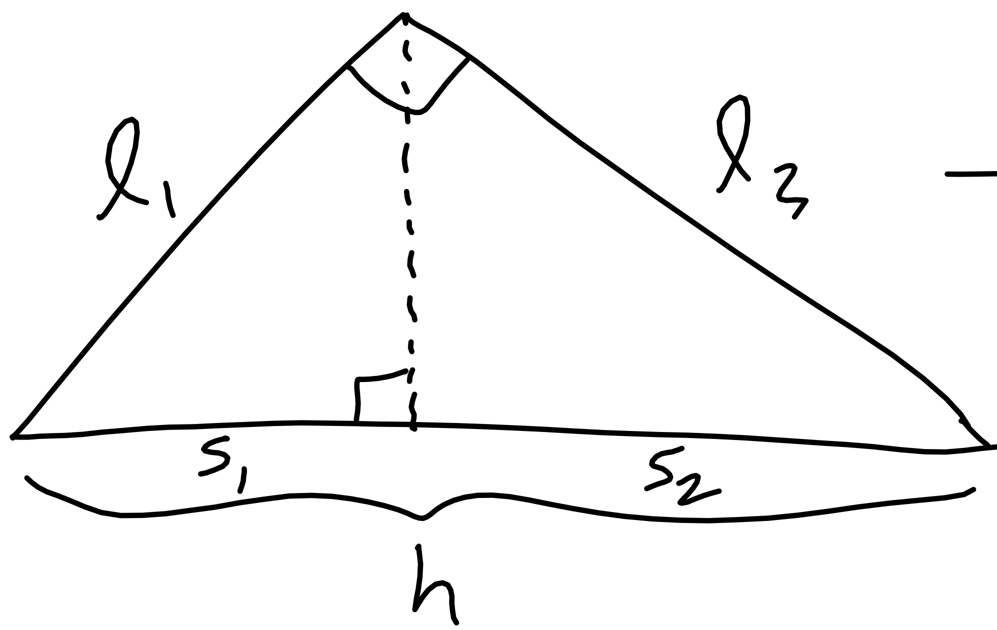



$(10)^2 = (\sqrt{5x})^2$

$100 = 5x$

$x = 20$

## - Corollary 2 (Legs Rule) →



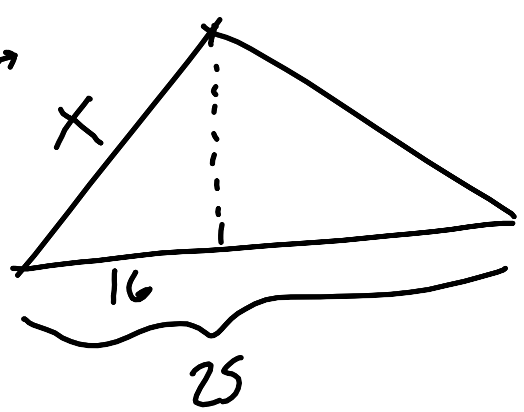

$$\frac{\text{leg}}{\text{hyp}} \qquad \frac{l_1}{h} = \frac{s_1}{l_1} \quad \text{OR} \quad \frac{l_2}{h} = \frac{s_2}{l_2}$$

↗ $l_1^2 = h \cdot s_1$

$l_1 = \sqrt{h \cdot s_1}$

(*) Leg length is geometric mean of hypotenuse & segment of hypotenuse adjacent to leg

EX →

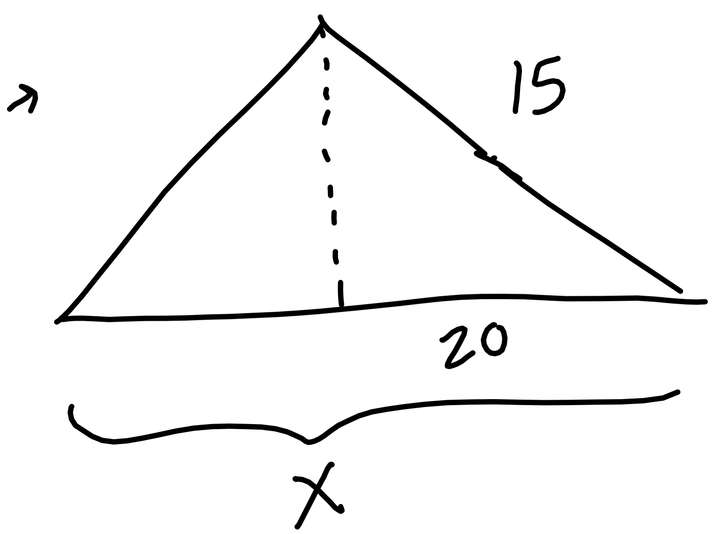



$X = \sqrt{25 \cdot 16}$

$X = \sqrt{400}$

$X = 20$

EX →



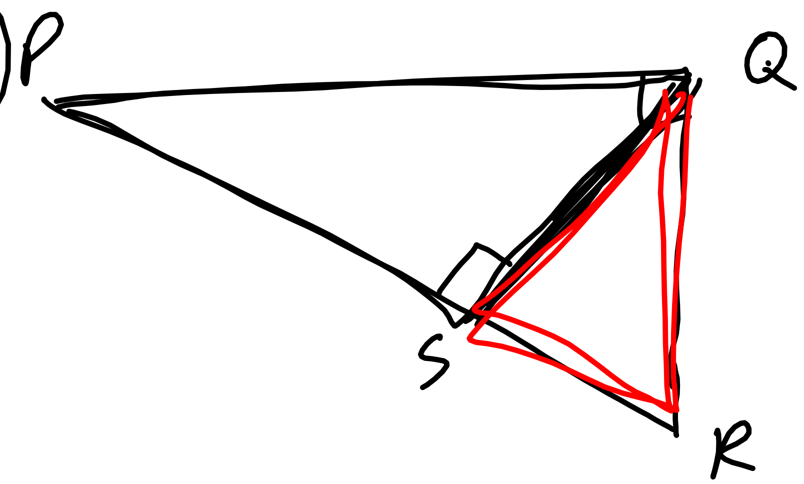

$15 = \sqrt{20 \cdot x}$

$15^2 = (\sqrt{20x})^2$

$20x = 225$

$X = 11.25$

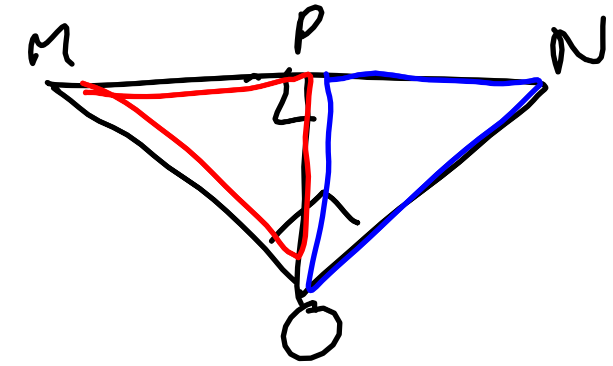
10




$\triangle PQR \sim \triangle PSQ \sim \triangle QSR$

small 90° big

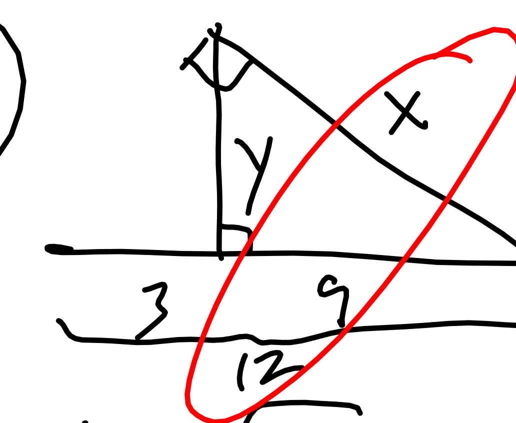
11




$\triangle MNO \sim \triangle MOP \sim \triangle ONP$

small big 90°

18




$y = \sqrt{9 \cdot 3} = 3\sqrt{3}$

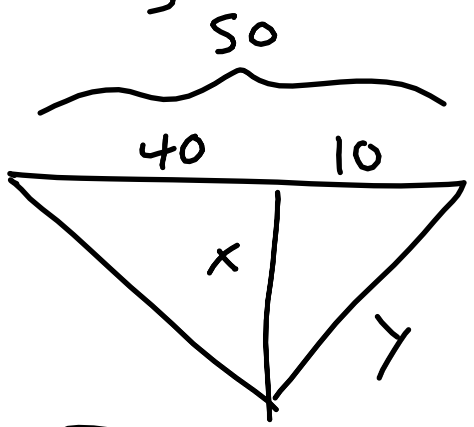
$\frac{\text{leg}}{\text{hyp}} = \frac{9}{x} = \frac{x}{12}$

$\sqrt{x^2} = \sqrt{108}$

$x = \sqrt{36} \cdot \sqrt{3}$

$x = 6\sqrt{3}$

19



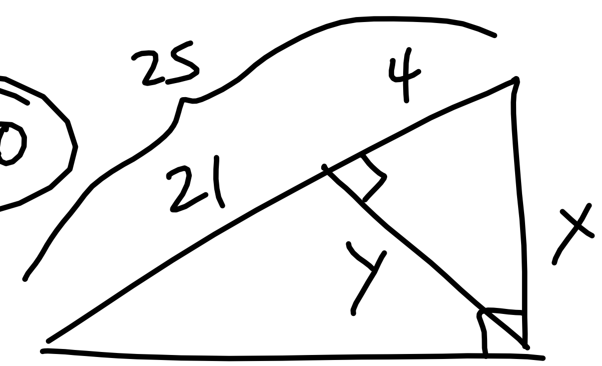

$x = \sqrt{40 \cdot 10} = \sqrt{400} = 20$

$\frac{10}{y} = \frac{y}{50}$ $\quad y^2 = 500$

$y = \sqrt{500}$

$y = \sqrt{100} \cdot \sqrt{5} = 10\sqrt{5}$

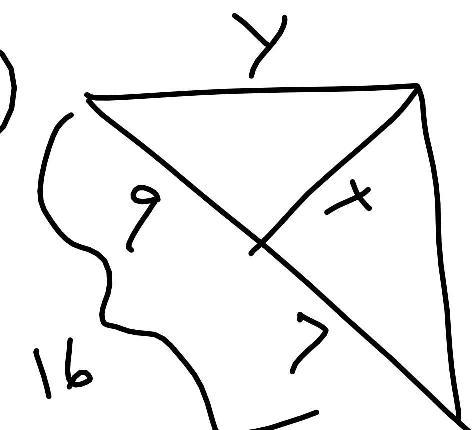
20




$y = \sqrt{21 \cdot 4} = 2\sqrt{21}$

$x = \sqrt{25 \cdot 4} = 10$

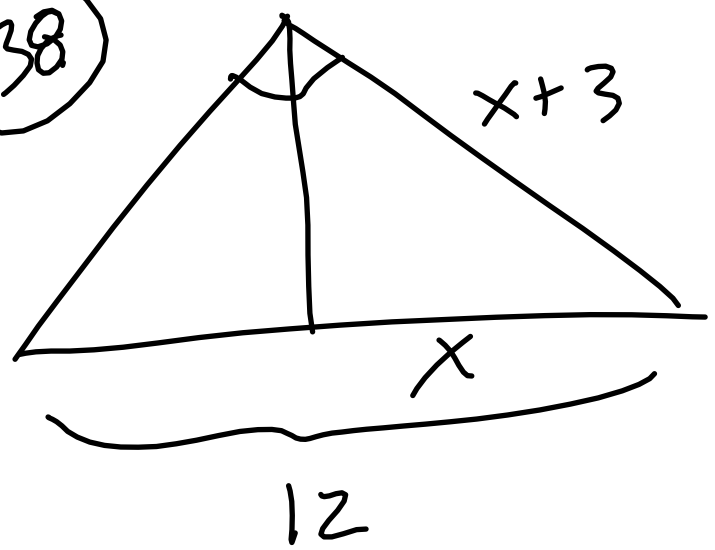
21




$x = \sqrt{9 \cdot 7} = 3\sqrt{7}$

$y = \sqrt{16 \cdot 9} = 12$

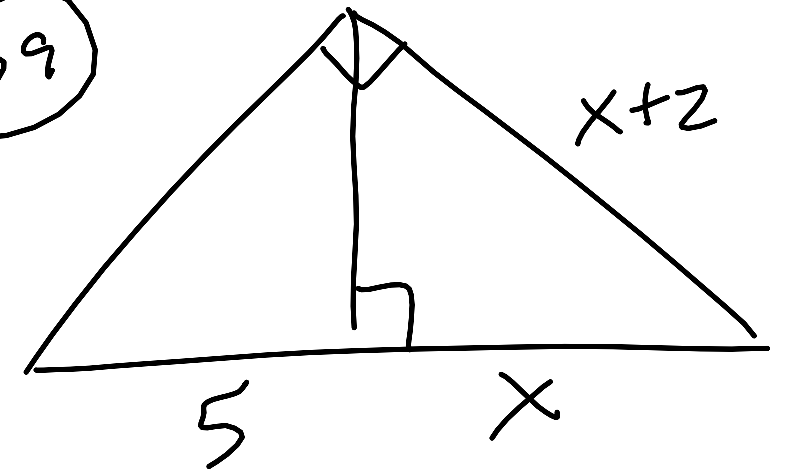
38




$$\frac{\text{leg}}{\text{hyp}} \quad \frac{x}{x+3} = \frac{x+3}{12}$$

$$(x+3)^2 = 12x$$
$$(x+3)(x+3) = 12x$$
$$x^2+6x+9 = 12x$$
$$x^2-6x+9=0$$
$$(x-3)(x-3)=0$$
$$(x-3)^2=0 \longrightarrow x-3=0$$
$$x=3$$

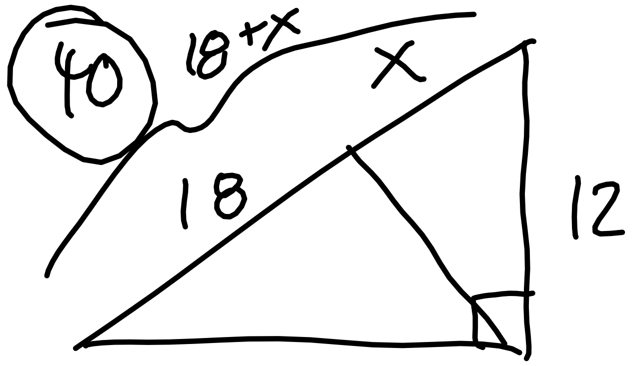
39




$$\frac{\text{leg}}{\text{hyp}} \quad \frac{x}{x+2} = \frac{x+2}{x+5}$$

$$(x+2)(x+2) = x(x+5)$$
$$\cancel{x^2}+4x+4 = \cancel{x^2}+5x$$
$$4=x$$

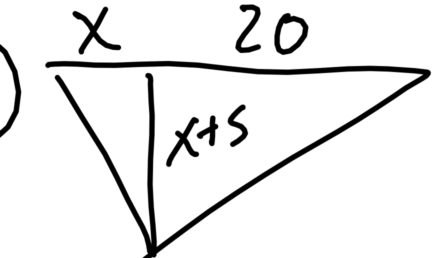
40




$$\frac{\text{leg}}{\text{hyp}} = \frac{x}{12} = \frac{12}{18+x}$$

$$x(x+18) = 144$$
$$x^2+18x = 144$$
$$x^2+18x-144=0$$
$$(x+24)(x-6)=0$$
$$x+24=0 \quad \underline{\text{OR}} \quad x-6=0$$
$$\cancel{x=-24} \qquad x=6$$

41




$$(x+5)^2 = \left(\sqrt{20\cdot x}\right)^2$$
$$(x+5)(x+5) = 20x$$
$$x^2+10x+25 = 20x$$
$$x^2-10x+25=0$$
$$(x-5)(x-5)=0$$
$$x-5=0$$
$$x=5$$

HW: p. 465 → 9-21, 24-28, 38-41, 48, 49