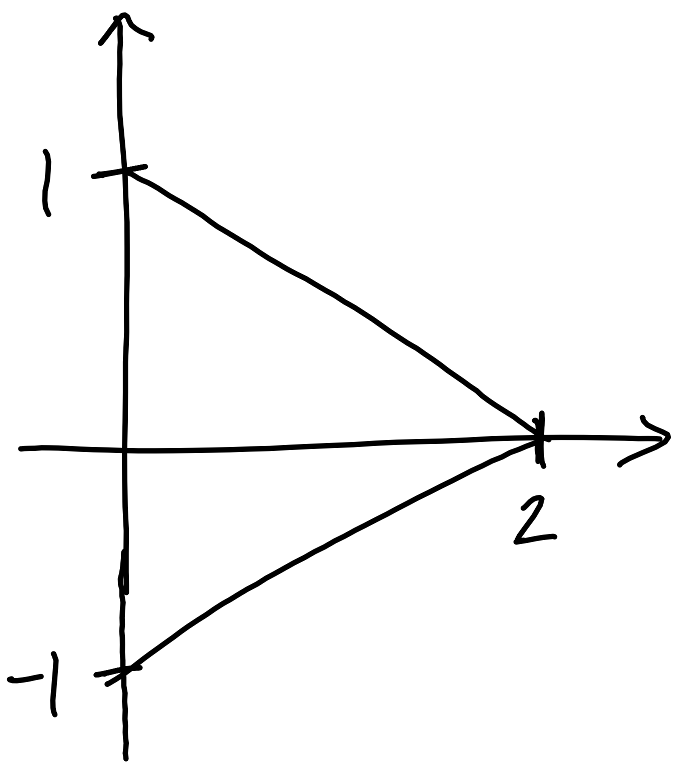
- With the disk & washer methods, we treated the solid of revolution like a cylinder. To evaluate, we took the integral of the area of a circle, which is the shape of a cross-section of a cylinder. Using that same reasoning, we can find the volume of any shape if we know the shape of the cross-section.
  - → If perpendicular to x-axis ⇒ $V = \int_a^b A(x)\,dx$
  - → If perpendicular to y-axis ⇒ $V = \int_a^b A(y)\,dy$
- Area Formulas to Know
  - → Square ⇒ $A = s^2$
  - → Rectangle ⇒ $A = l \cdot w$
  - → Triangle ⇒ $A = \frac{1}{2}bh$
  - → Equilateral Triangle ⇒ $A = \frac{\sqrt{3}}{4}s^2$
  - → Trapezoid ⇒ $A = \frac{1}{2}(b_1 + b_2)h$
  - → Ellipse ⇒ $A = \pi r_1 r_2$

-EX1 → Find the volume of the figure w/ an area b/w $y = 1 - \frac{x}{2}$, $y = -1 + \frac{x}{2}$, and $x = 0$, where the cross-sections are equilateral triangles



$$V = \int_0^2 \frac{\sqrt{3}}{4}\left(1 - \frac{x}{2} - \left(-1 + \frac{x}{2}\right)\right)^2 dx$$

$$= \int_0^2 \frac{\sqrt{3}}{4}(2-x)^2 dx = \frac{\sqrt{3}}{4}\int_0^2 4 - 4x + x^2 \, dx$$

$$= \frac{\sqrt{3}}{4}\left(4x - 2x^2 + \frac{1}{3}x^3 \Big|_0^2\right) = \frac{\sqrt{3}}{4}\left[4(2) - 2(2)^2 + \frac{1}{3}(2)^3 - 0\right]$$

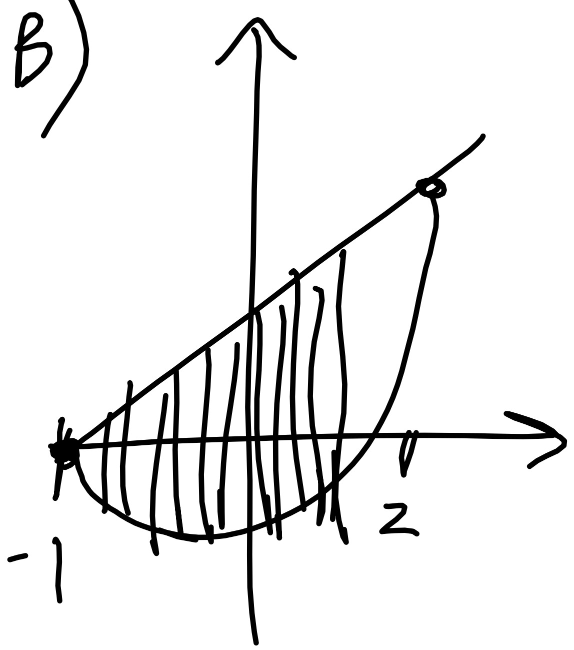
$$= \frac{\sqrt{3}}{4}\left[\cancel{8 - 8}^{0} + \frac{8}{3}\right] = \frac{8\sqrt{3}}{12} = \boxed{\frac{2\sqrt{3}}{3} u^3}$$

- EX2 → Using the same equations from EX1, find the volume where the cross-sections are rectangles w/ a height of 2.

$$V = \int_0^2 (2-x)(2)\, dx = \int_0^2 4 - 2x \, dx = 4x - x^2 \Big|_0^2 = \left[4(2) - 2^2\right] - 0 = 8 - 4 = 4$$

<u>HW</u>: p. 465 → 61-63, 75

61 B)



$$V = \int_{-1}^{2} \left((x+1)-(x^2-1)\right)(1)\,dx$$

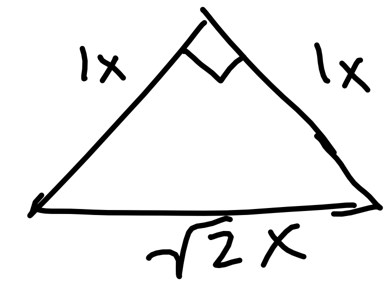
$$= \int_{-1}^{2} x - x^2 + 2\,dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x \Big|_{-1}^{2}$$

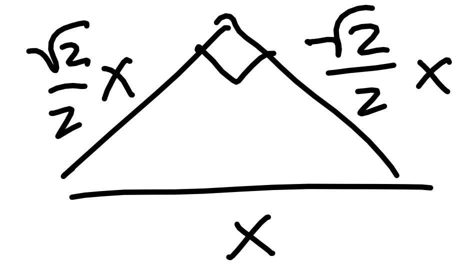
$$= \left[\frac{1}{2}(2)^2 - \frac{1}{3}(2)^3 + 2(2)\right] - \left[\frac{1}{2}(-1)^2 - \frac{1}{3}(-1)^3 + 2(-1)\right]$$

$$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right) = \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

62 D)

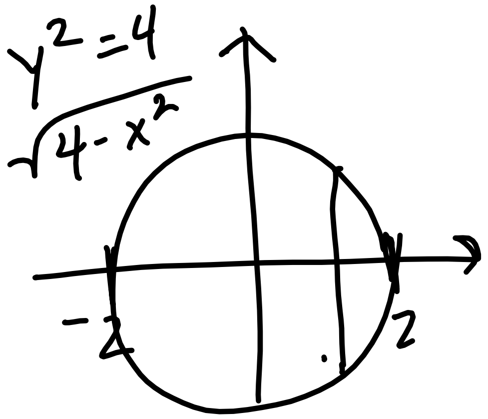


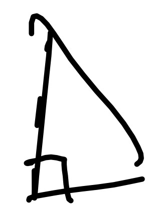
$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



$x^2 + y^2 = 4$

$y = \pm\sqrt{4 - x^2}$





$$V = \int_{-2}^{2} \frac{1}{2}\left(\frac{\sqrt{2}}{2}\cdot 2\sqrt{4-x^2}\right)\left(\frac{\sqrt{2}}{2}\cdot 2\sqrt{4-x^2}\right)dx$$

$$= \int_{-2}^{2} \frac{1}{2}\left(2\cdot(4-x^2)\right)dx$$

$$= \int_{-2}^{2} 4 - x^2\,dx$$

$$= 4x - \frac{1}{3}x^3 \Big|_{-2}^{2}$$

$$= \left[4(2) - \frac{1}{3}(2)^3\right] - \left[4(-2) - \frac{1}{3}(-2)^3\right]$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= \frac{16}{3} - \left(-\frac{16}{3}\right) = \frac{32}{3}$$